

THE CONSTRUCTION OF THE BEST MATHEMATICAL MODEL FOR CALCULATING RADON CONCENTRATION IN MINERAL WATER

Ghassan Ezzulddin ARIF¹

Tikrit University, Iraq

Abstract

In this paper, we presented three mathematical models to calculate the amount of radon concentration in water which is considered a poisonous gas. two of these models are established by numerical methods, which are the Neville method and the Spline method. The graph approach is used to create the third model. It is notable that the current study is original and having a novelty as it has created for the first time three mathematical methods to calculate radon gas concentrations in water; as well as it is the first study that has applied the numerical methods to carry out the calculations. the results obtained from these methods are compared with previous studies and then finding the error rates. After that, we have made a comparison between these three models to obtain the best one. Since the error rate is slightly lower than the Neville method so the Spline method is the best of the three methods that we worked with.

Keywords: Radon Concentration, Mathematical Model, Neville Method, Spline Method, Graph Method.

Introduction

Inhalation of breakdown products delivered by radon gas poses a significant risk to human health. To a person's lungs, aerosols. The alpha particle emissions from decay products dominate the radiation dose for lung tissue, which then attach to the lung and cause damage. It damages sensitive cells and raises the risk of cancer [7]. When radon is allowed to accumulate in a building, for example in inner radon vent tube, where it penetrates the radiation of alpha particles which may result in negative and serious effects on human health. That's why certain sections of the population in increased risk of suffering from harm to health from radon [3]. Also exposure is high gas levels also be cause it to happen cancers, such as leukemia in children. Long-term exposure to radiation, in any form, harms genetic and human fertility and activity, severely influencing future generations [2]. In our current study, we took a sample of mineral water from ten different regions, then we created three different models to determine the amount of radon in this water and compared these models to obtain the best model for determining the amount of radon in mineral water.

2- Preliminaries:

Definition 2.1 [4] Let f be a function that has been defined at $x_0, x_1, x_2, \dots, x_n$, as well as suppose m_1, m_2, \dots, m_k are k distinct integers, with $0 \leq m_i \leq n$ for each i . At the k points, the Lagrange polynomial that agrees with $f(x)$ at $x_{m_1}, x_{m_2}, \dots, x_{m_k}$ is denoted $P_{m_1, m_2, \dots, m_k}(x)$.

Theorem 2.2 [8] Let f be a function that has been defined at $x_0, x_1, x_2, \dots, x_k$, and let x_j and x_i be two distinct numbers in this set.

Then

$$P(x) = \frac{(x - x_j)P_{0,1,\dots,j-1,j+1,\dots,k}(x) - (x - x_i)P_{0,1,\dots,j-1,j+1,\dots,k}(x)}{(x_i - x_j)}$$

is the k th polynomial of Lagrange that interpolates f at the $k + 1$ points $x_0, x_1, x_2, \dots, x_k$. Neville's approach is a recursive procedure that exploits the result of Theorem 2.2 to construct interpolating polynomial approximations.

2-3- Algorithm of Neville's method [5,6,11]. Neville's method's algorithm is as follows:

Step 1: For $i = 1, 2, \dots, n$
 for $j = 1, 2, \dots, i$

$$Q_{i,j} = \frac{(x - x_{i-j})Q_{i,j-1}(x) - (x - x_{i-j})Q_{i-1,j-1}(x)}{(x_i - x_{i-j})}$$

Step 2: OUTPUT (Q) ;
 STOP.

Section 3 Create three mathematical models to determine the percentage of radon in mineral water

In this section, we will present three different mathematical models to determine the amount of radon in mineral water, two of which are numerical and the third dependent on the graph, and the comparison of the three mathematical models to determine the best among them. We'll start with a Spline method of ten different amounts of mineral water.

3.1 - Formula of concentration of lead radioactivity CRn in Radon (Spline method)[12]

X = concentration of lead in mineral water samples
Y = annual dose radioactivity

Then this equation given as following:

$$y = y_0 + \frac{y_1 - y_0}{x_1 - x_0} (x - x_0)$$

$$x_0 = 76.42 \quad y_0 = 0.27$$

$$x_1 = 91.22 \quad y_1 = 0.32$$

$$= 0.27 + \frac{0.32 - 0.27}{91.22 - 76.42} (x - 76.42)$$

$$= 0.27 + \frac{0.05}{14.8} (x - 76.42)$$

$$= 0.27 + 0.0033783784 (x - 76.42)$$

$$= 0.27 + 0.0033783784x - 0.2581756773$$

$$= 0.0118243227 + 0.0033783784x .$$

Table 1. Using the spline method to determine y Det. and comparing it to the y Exp. values.

No.	X	y Exp.	y Det.	Exp-Det
1	76.42	0.27	0.27	0.00
2	91.22	0.32	0.32	0.00
3	98.61	0.35	0.344966	0.005034
4	101.08	0.36	0.353311	0.006689
5	175.04	0.63	0.603176	0.026824
6	197.23	0.71	0.678142	0.031858
7	234.22	0.84	0.803108	0.036892
8	253.94	0.91	0.86973	0.04027
9	263.80	0.95	0.903041	0.046959
10	350.09	1.26	1.194561	0.065439

Y Exp. represents experimental values; y Det. refers to obtained theoretical result;
Exp-Det is the error rate between y. Exp. and Y. Det.

Figure 1. indicates the comparison between y. Det. with y. Exp. values by using spline method.

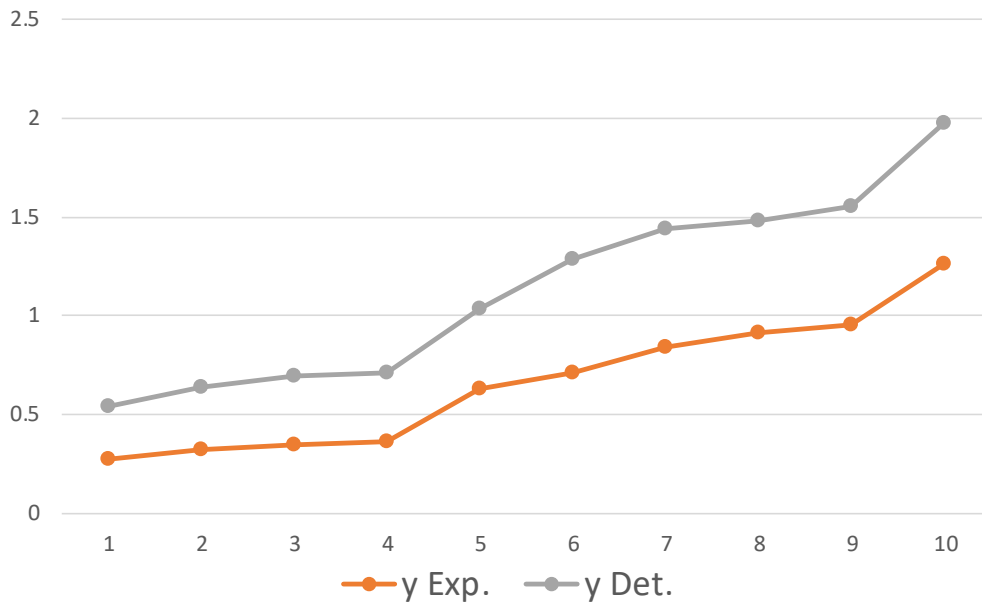


Fig. 1: Comparison between y. Det. with y. Exp. values.

3.2 - Formula of concentration of lead radioactivity CRn in Radon (Neville method)

X = annual Concentration of lead radioactivity CRn in mineral water samples = Y.
dose

Then this equation given as following :

$$\begin{aligned}
 y &= \frac{(x - x_0)y_1 - (x - x_1)y_0}{(x_1 - x_0)} \\
 x_0 &= 76.42 & y_0 &= 0.27 \\
 x_1 &= 91.22 & y_1 &= 0.32 \\
 &= \frac{(x - 76.42)0.32 - (x - 91.22)0.27}{(91.22 - 76.42)} \\
 &= \frac{0.32x - 24.4544 - 0.27x + 24.6294}{14.8} \\
 &= \frac{0.05x + 0.175}{14.8} \\
 &= 0.0033783784 x + 0.0118243243
 \end{aligned}$$

Table 2. Using the Neville method to determine y Det. and comparing it to the y Exp. values.

No.	X	y Exp.	y Det.	Exp-Det
1	76.42	0.27	0.27	0.00
2	91.22	0.32	0.32	0.00
3	98.61	0.35	0.344966	0.00503378
4	101.08	0.36	0.353311	0.00668919
5	175.04	0.63	0.603176	0.02682432
6	197.23	0.71	0.678142	0.0318581
7	234.22	0.84	0.803108	0.03689189
8	253.94	0.91	0.86973	0.04027026
9	263.80	0.95	0.903041	0.04695945
10	350.09	1.26	1.194561	0.06543918

Figure 2. indicates the comparison between y. Det. with y. Exp. values by using Neville method.

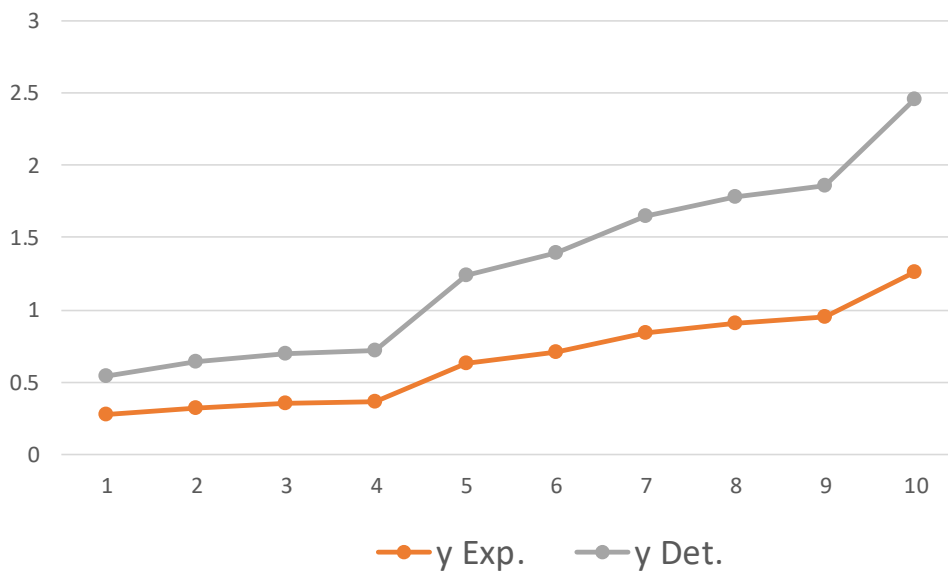


Fig. 2: Comparison between y. Det. with y. Exp. values.

Now ,we will present the graph to determine the amount of radon in mineral water [9-10].

3.3- Formula of concentration of lead radioactivity CRn : (Graphical method)

where x_1, x_2, \dots, x_{10} is vertices in graph
and X = annual dose Concentration of lead

radioactivity CRn in mineral water samples= Y. Then this equation given as following:

$$\begin{aligned}
 S(X) &= \frac{(X-x_1)y_{10}-(X-x_2)y_2}{(X_{10}-X_2)} \\
 &= \frac{(X-76.42)1.26-(X-91.22)0.32}{350.09-91.22} \\
 &= \frac{1.26X-96.2892-0.32X+29.1904}{273.67} \\
 &= \frac{0.94X-0.67.0988}{273.67} \\
 S(X) &= 0.0034348X-0.24518142 .
 \end{aligned}$$

Table 3. Using the Graphical method to determine y Det. and comparing it to the y Exp. values.

No.	X	y Exp.	y Det.	Exp-Det
1	76.42	0.27	0.017306	0.252694
2	91.22	0.32	0.06814104	0.25185896
3	98.61	0.35	0.09352421	0.25647579
4	101.08	0.36	0.10200816	0.25799184
5	175.04	0.63	0.35604597	0.27395403
6	197.23	0.71	0.43226418	0.27773582
7	234.22	0.84	0.55931744	0.28068256
8	253.94	0.91	0.62705169	0.28294831
9	263.80	0.95	0.66091882	0.28908118
10	350.09	1.26	0.95730771	0.30269229

Figure 3. indicates the comparison between y. Det. with y. Exp. values by using Graphical method.

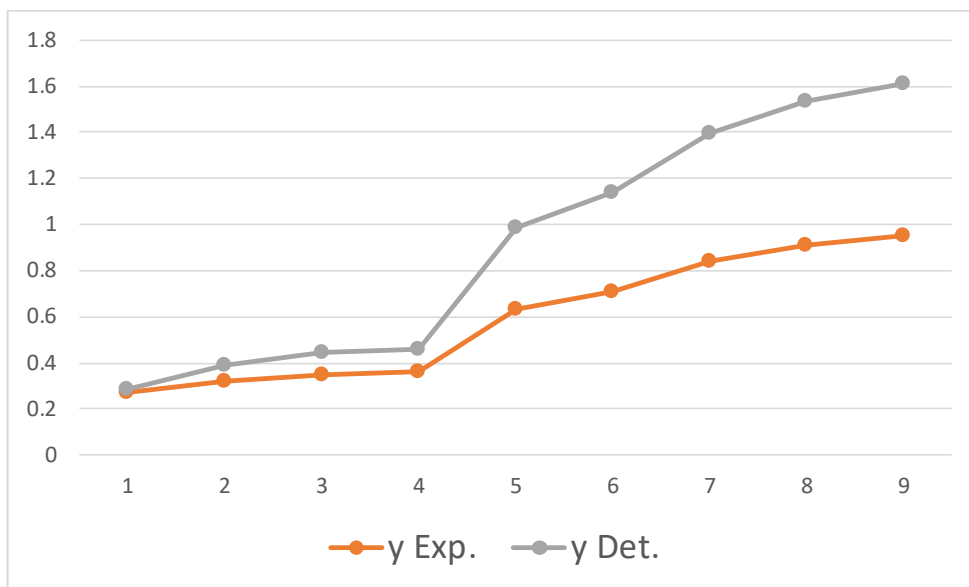
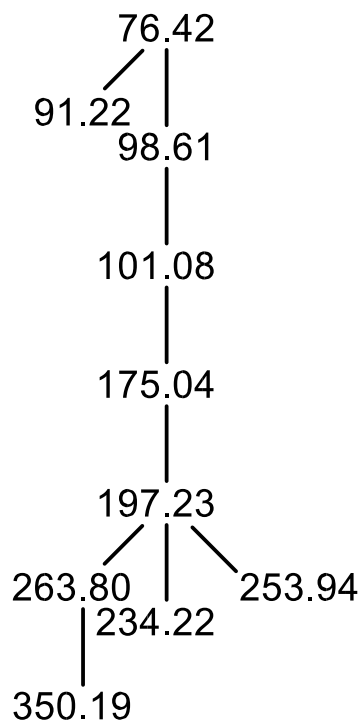


Fig. 3: Comparison between y. Det. with y. Exp. values.

Now, we will give the flowchart to the graphic model as following:



Flowchart 3.4 y via the x_i , then using graph theory to compare them to the y Exp. values.

4- Conclusion:

The following was concluded in this paper:

We discovered that the best way to determine the proportion of radon in mineral water is the Spline method in a sample of 10 different volumes of mineral water because the error rate is significantly lower than the Neville technique, and the drawing approach has a much lower error rate. The Spline method is the best of the three methods that we worked with because the error rate is less by a difference that is not a little bit in the way of drawing, whereas the Neville method has a difference that is not A little bit in the way of drawing, so the Spline method is the best of the three methods that we worked with.

References:

- 1-** Bracewell, R., *The Fourier transform and its application*, (Third edition), McGrawHill, New York, 616 pp. QA403.5.B7, (2000).
- 2-** Campi, F., Caresana, M., Ferrarini, M., Garlati, L., Palermo, M. and Rusconi, R., "Uncertainty evaluation of radon measurements with LR115 detector and spark counter", *Rad. Prot. Dos.*, 111:59-64, (2004).
- 3-** Laich T. P. "A radiological evaluation of phosphogypsum", *Health Phys.*, 60 691-693, (1991).
- 4-** Saff, E. B. and A. D. Snider, *Fundamentals of complex analysis for mathematics, science, and engineering*, (Third edition), Prentice-Hall, Upper Saddle River, NJ, 511 pp. QA300.S18, (2003) .
- 5 -**Steele, J. Michael, *The Cauchy-Schwarz Master Class*. Cambridge University Press, 306 pp. QA295.S78,(2004).
- 6-** Strikwerda, J. C., *Finite difference schemes and partial differential equations*, (second Edition), SIAM Publications, Philadelphia, PA, 435 pp. QA374.S88 761,(2004) .
- 7-** United Nations Scientific Committee "on the Effects of Atomic Radiation UNSCEAR 2000 report to the general assembly", with scientific annexes. Sources and effects of ionizing radiation. United Nations New York, (2000) .
- 8-** Varga, R. S., "Matrix iterative analysis", (Second edition), Springer, NewYork, , 358 pp. QA263.V3 495, 718-722,(2000).
- 9-** Shamim Ahmed, Applications of Graph Coloring in modern computer science, *Int. J. of Comp. Inf. Tech.*,3(2) 1-7 (2012).
- 10-** Shirinivas, S.G. , Vetrivel, S. and Elango, M, Applications of Graph Theory in computer Science review, *Int. J. of Engg. Sci. and Tech.*, 2(9) 4610-4621(2010).
- 11-** Arif, Ghassan E., Samar W. Omar, and Aws A. Hamdi. "Establishing mathematical models to estimate the activities of radionuclides (ra) and the lung cancer (lc) by using some numerical method." *AIP Conference Proceedings*. Vol. 2213. No. 1. AIP Publishing LLC, 2020.
- 12-** Hamdi, Aws Asaad, Ghassan Ezzulddin Arif, and Anas Asaad Hamdi. "Establishing mathematical models for determining quantum dot potential for pbs and pbte structures in terms of QD's diameters by numerical methods." *International Journal of Nonlinear Analysis and Applications* 13.1 (2022): 729-735.