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FRICTIONAL EFFECTS ON THE DYNAMIC RESPONSES OF A SINGLE-STAGE SPUR GEAR SYSTEMS

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Abstract

Gears are paramout rotary mechanical equipment as its used in many industrial applications, for example in cars, industrial compressors and other applications. Therefore, monitoring the development of its condition is very important to prevent the aggravation of defects and stopping production. Friction between gears is causes of vibration but its representation in modeling and the analysis of its effect on the dynamic response is a very complex matter. In this study, the friction effect was studied by relying on equal load sharing formula and by relying on the sliding velocity direction of single-stage spur gears. The time domain was converted to the frequency domain, depending on the Fast Fourier Transform (FFT) method. Dynamic modeling results indicate that the friction between gears has a significant effect on the Vibration response of gearbox. This effect can be noticed by increasing the vibration amplitude in the time domain. It can also be seen by increasing the gear mesh frequency (GMF) amplitude and by increasing the amplitude of its harmonics in the frequency domain.

Keywords: Spur Gear, Fast Fourier Transform (FFT), Friction, Mesh Stiffness, Sliding Velocity, Vibration Response.

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1. Introduction

At last years, gears have become very important in transmitting mechanical power at different speeds and loads with high efficiency. Therefore, most researchers have tended to search for methods that can monitor gear conditions [1][2]. They concluded that there are different methods that can monitor gear conditions such as Motor Current Signature Analysis, Acoustic Emission Analysis, Vibration Signal Analysis, Temperature Analysis and other methods [3]. The advantages of the Condition Monitoring Techniques are: First, they reduce downtime. Second, it reduces maintenance costs and financial losses. Third, by predicting the occurrence of faults in time, this method allows for proper maintenance [4]. In addition, this method allows the employment of the equipment’s service life for the longest possible period [5][6]. Currently, condition monitoring (CM) techniques are widely applied, particularly the commonly used techniques, and the aim of their application is to keep the gears in good condition [7][8]. Vibration analysis technology is the best way to monitor gear conditions[9]. Due to the complexity of the gearboxsystems, it is imperative to understand the causes of the vibrations and noise that occur in gears while they are turning [10]. Many researchers have developed dynamic models to analyze gear vibration [11][12][13][14][15]. Some of these papers neglected the friction effect, while others assumed that the friction effect was constant. This thing is incorrect because the effect of friction is considered to be variable with time and greatly influences translational responses. Therefore, friction must be properly and appropriately represented to increase the ability to accurately diagnose gear defects. The source of vibration between the teeth while attaching gears is friction. Friction dissipates the energy transmitted through the gears, resulting in plastic deformation, which increases the wear of the gears and increases the conceivable of gear failure and system breakdown [10]. A. Kahraman et al proposed several empirical formulas to computation the friction coefficient and to study its effect. These formulas are based on load ranges, specific lubricants and operating temperatures which vary according to the conditions in which the gears operate. It was found that these formulas couldnot be exhaustive because it was valid for specific ranges [16]. Rajendra Singh et al added friction to study gear design modifications [17].

This paper analyzes vibration (resonance frequencies) and studies the friction effect on the responses of single-stage spur gears.

2. Gear system modeling

2.1 Mesh stiffness

The mesh stiffness (MS) is an important parameter by which it is possible to represent the gears defects and study its effect on the responses of the gears [18][19][20]. The number of pairs involved in the meshing of spur gears varies with time so the MS value relying on the number of pairs present in the meshing process.

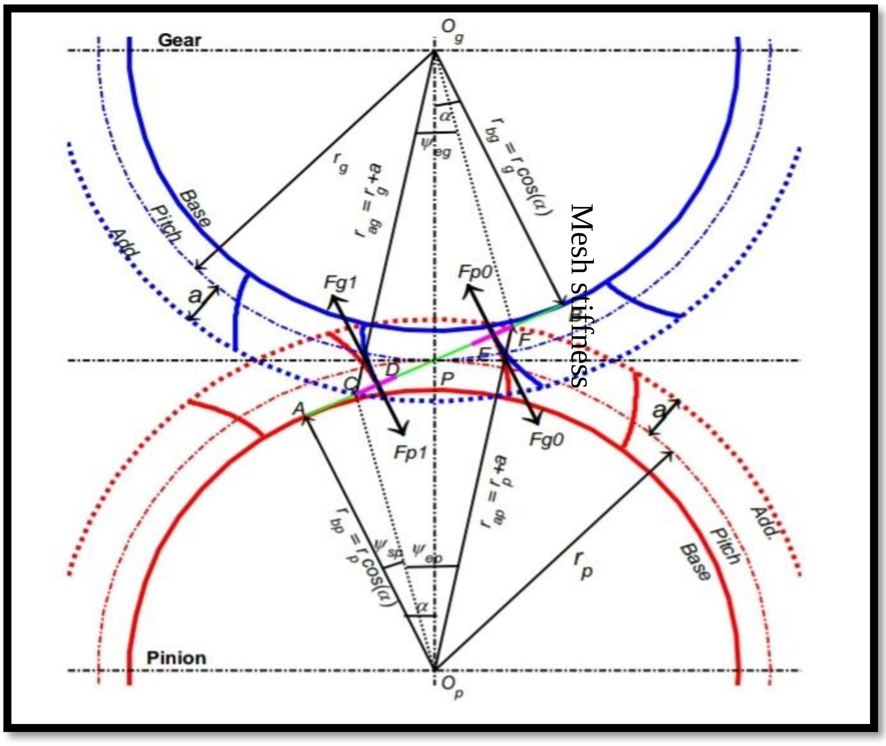


Figure 1. Meshing process of spur gear pairs [10].

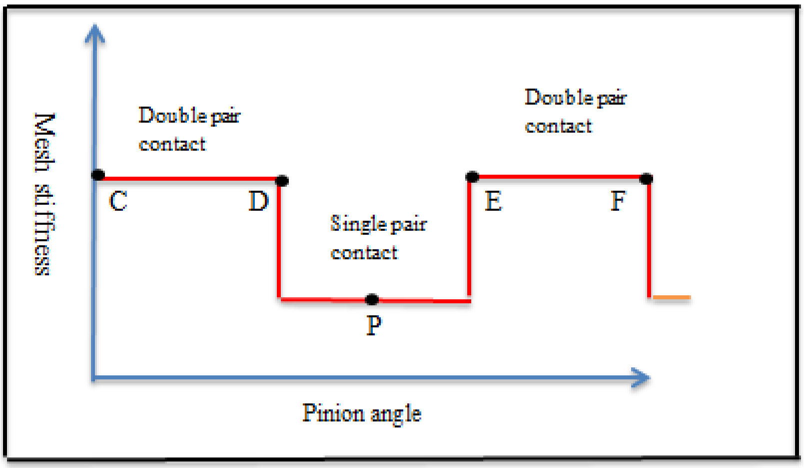


Figure 2. regions of gear meshing stiffness change.

The lengths in Figure 1 can be computation as follows:

$$AC = (r_p + r_g)\sin(\alpha) - \sqrt{r_{ag}^2 - r_{bg}^2} \qquad (1)$$

$$FB = (r_p + r_g)\,sin(\alpha) - \sqrt{r_{ap}^2 - r_{bp}^2} \qquad (2)$$

$$DF = CF = \frac{2\pi\, r_{bp}}{z_1} \qquad (3)$$

Figure 1 illustrates the meshing process of the spur gears. In this study, the MS was added depending on the equal load sharing formula suggested by Valshya and Singh [21]. The value of the maximum MS for one pair of teeth is 107 N / m and when there are two pairs in the meshing process, this value is doubled as shown in Figure 2 [3].

2.2 Friction

Friction is one of the sources of vibration (see Section 1) so it must be represented within a model to obtain more accurate results. Representing friction with a constant value may give acceptable results, but the representation of friction depending on the sliding velocity direction (Vs) is the best way to represent it. sliding velocity can be computable by relying on the friction torque arms and on the angular velocity of gear and pinion as shown in Equation 6. However, the constant coefficient of friction(μo) gives acceptable results, and it is most likely to be between 0.03 to 0.2. In this study, the friction coefficient (μ) was represented by relying on the equal load sharing formula applied in [21]. μo of 0.1 was used, but it changes with the change in the Vsdirection according to Equation 7.

Friction forces were calculated as follows:

$$\rho_p(t) = AC + mod(r_{bp}\,\omega_p, CE)(4)$$

$$\rho_g(t) = FB + mod(r_{bg}\omega_g, CE)(5)$$

$$V_s = \rho_{pi}(t)\omega_p - \rho_{gi}(t)\omega_g \qquad (6)$$

$$\mu = \mu_o sgn(V_s) \qquad (7)$$

$$N_i = K_m(t)(r_{bp}\theta_p - r_{bg}\theta_g + y_p - y_g) + C_m(t)(r_{bp}\dot{\theta}_p - r_{bg}\dot{\theta}_g + \dot{y}_p - \dot{y}_g)(8)$$

$$Ff(t) = \mu N_i(9)$$

Moment friction is calculated as follows:

$$T_p(t) = Ff(t)\rho_p(t) \qquad (10)$$

$$T_g(t) = Ff(t)\rho_g(t) \qquad (11)$$

where i=0, 1 denoting entanglement pair of tooth. ρg , ρp represents the friction arm value of gear and pinion, respectively.ωg , ωprepresents the angular velocity of gear and pinion, respectively.
function mod represents a function of modulus and can be represented as follows:

$$mod\ (x,y) = x - y \cdot floor\left(\frac{x}{y}\right)$$

if y≠0, ωp and ωg are the angular velocity in (rad/s), and AC and FB are the geometric length constants [21]. The calculation of the damping coefficient has been linked with the MSvalue [22] as follows:

$$C_m(t) = 2\zeta\sqrt{K_m(t)\left(I_p + I_g\right)} \qquad (12)$$

Figure 3.a shows the value of the total friction forces as the total friction forces at the pitch point (P) are equal to zero because the meshing process at point P becomes a rolling process, so there is no friction at this point and the same applies to Figure 3.b, which shows the torque of friction where the total friction torque has the point P is equal to zero because the forces of total friction at point P are equal to zero.

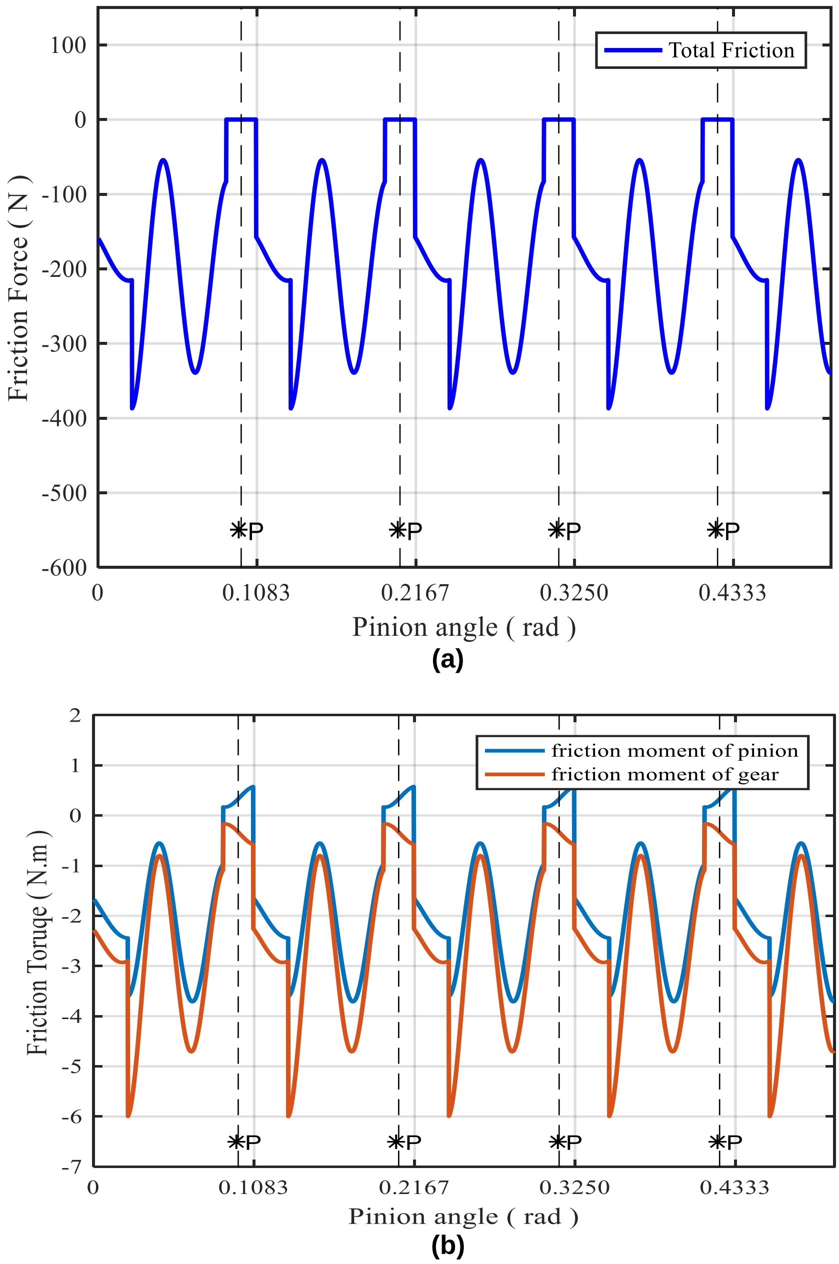


Figure 3. Variation of friction forces(a) and frictional torque (b) with the pitch period.

2.3 Dynamic model of gears

At last years, the modeling has achieved remarkable results in analyzing the dynamics of gear action and in detecting and diagnosing defects of gearbox[23][24][25]. Modeling is done in multiple ways, such as modeling in a finite element method , through dynamic modeling by Matlab program or through other methods. The mathematical model suggested by Muhammad [14]was used which consists of six freedom degrees (4 translations 2 rotations).

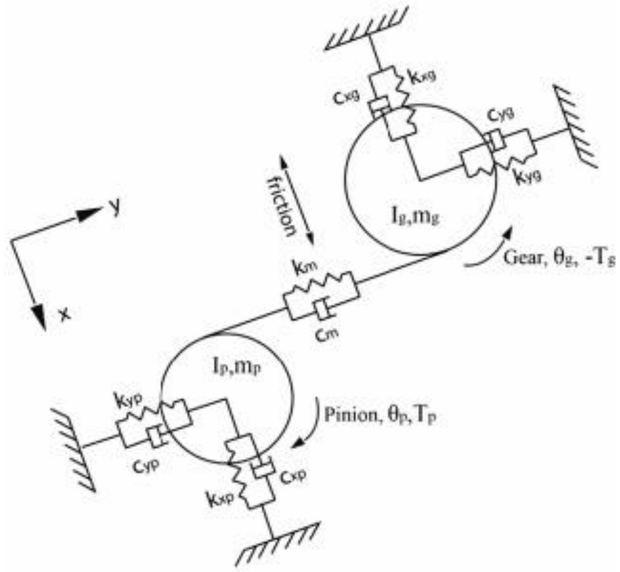


Figure 4. Single-stage spur gear model.

This mathematical model consists of two inertia, namely gear and pinion. It also consists (see Fig. 4) of a time-varying meshing stiffness Km(t) and a time-varying damping meshing Cm(t) through which the pinion are connected to the gear. motor inertia and load values were neglected becausethey did not affect the results[14]. The pinion and gear were assumed to be solid discs with manufacturing and design errors neglected. The main parameters of the gearsare shown in Table 1.

Table 1. Main parameters of the gear transmission system.

Parameters	Pinion	Gear
Teeth number	ZP = 58	Zg =47
Pitch radius	40.1(mm)	32.5(mm)
Mass (kg)	0.9	0.58
Rotation speed	1485(rpm)	1832.6(rpm)
Pressureangle	20(°)	
Addendum (mm)	1.4	
Contact ratio	1.7822	
Motor torque (N.m)	36	
Applied torque (N.m)	29.2	

According to Newton's law the equations of motions are computable as follows:

$$I_p\ddot{\theta}_p + C_m(t)r_{bp}\left(r_{bp}\dot{\theta}_p - r_{bg}\dot{\theta}_g + \dot{y}_p - \dot{y}_g\right) + K_m(t)r_{bp}\left(r_{bp}\theta_p - r_{bg}\theta_g + y_p - y_g\right) + T_{fp}(t) = T_p \quad (13)$$

$$I_g\ddot{\theta}_g - C_m(t)r_{bg}\left(r_{bp}\dot{\theta}_p - r_{bg}\dot{\theta}_g + \dot{y}_p - \dot{y}_g\right) - K_m(t)r_{bg}\left(r_{bp}\theta_p - r_{bg}\theta_g + y_p - y_g\right) + T_{fg}(t) = -T_g \quad (14)$$

$$m_p\ddot{y}_p + C_m(t)\left(r_{bp}\dot{\theta}_p - r_{bg}\dot{\theta}_g + \dot{y}_p - \dot{y}_g\right) + K_m(t)\left(r_{bp}\theta_p - r_{bg}\theta_g + y_p - y_g\right) + C_{by1}\dot{y}_p + K_{by1}y_p = 0 \quad (15)$$

$$m_g\ddot{y}_g - C_m(t)\left(r_{bp}\dot{\theta}_p - r_{bg}\dot{\theta}_g + \dot{y}_p - \dot{y}_g\right) - K_m(t)\left(r_{bp}\theta_p - r_{bg}\theta_g + y_p - y_g\right) + C_{by2}\dot{y}_g + K_{by2}y_g = 0 \quad (16)$$

$$m_p\ddot{x}_p + C_{bx1}\dot{y}_p + K_{bx1}y_p - F_{f12} = 0 \quad (17)$$

$$m_g\ddot{x}_g + C_{bx2}\dot{x}_g + K_{bx2}x_g + F_{f12} = 0 \quad (18)$$

$$T_p = T_p + 10(\omega_p - \dot{\theta}_p) \quad (19)$$

These equations are solved in Matlab based on ode15s solver. The reason to using the last equation is to stabilize the rotational velocity as much as possible. The following notation in the figure 4 :
mp /mg : Mass of the pinion/gear
Ip/Ig : Inertia mass moment of pinion/gear
Kxp/Kyp : Radial stiffness in the x/y directions of pinion
Kxg/ Kyg: Radial stiffness in the x/y directions of gear
Cxp/Cyp : Radial damping in the x/y directions of pinion
Cxg/Cyg : Radial damping in the x/y directions of gear
Km : Equivalent mesh stiffness
Cm : Mesh damping coefficient
Tp/Tg : Torque applied on the pinion/gear
rbp/rbg : Base circle radius of the pinion/gear

3. Results

3.1 Linear Solution

A simplified linear model was developed based on the average value of the MS in (13) - (18) depending on the state space method. This method allows modal parameters for example damping ratios and resonance frequencies to find appropriately. Monitoring resonant frequencies is very necessary because it accelerates system failure by amplifying gear vibration to levels that exceed the permissible limit of gear design [26].
The differential equation for vibration is expressed as follows:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = f(t) \quad (20)$$

$$\{\dot{V}\} = [A]\{q\} \quad (21)$$

Where [C] is the matrix of damping, [M] is the matrix of mass, [K] is the matrix of stiffness and q is the vibration response vector consisting of the displacement and the velocity of the system.

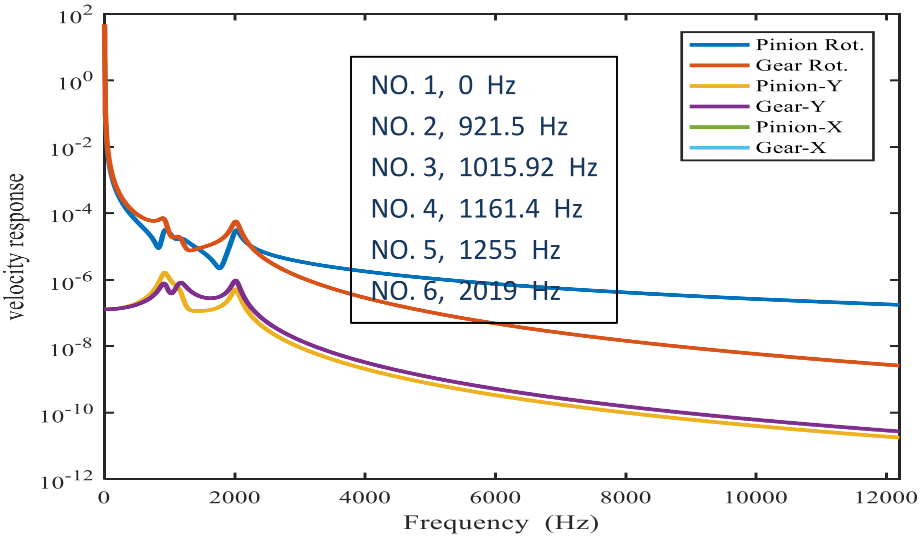


Figure 5. Velocity response to meshing excitation.

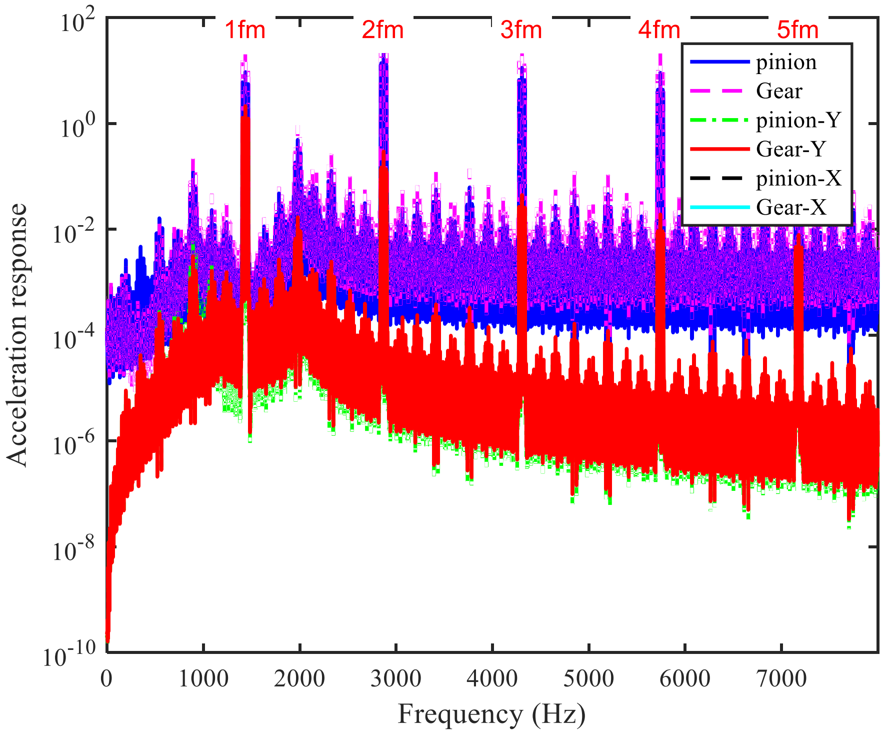


Figure 6. Acceleration response to meshing excitation.

Figure 6 and Figure 5 illustrate the acceleration and velocity response, respectively. It can be noticedthat the first mode is at 921.5Hz which is 37 times away from shaft frequency at about 25Hz.
The fifth mode (1255 Hz) is close to GMF. The friction effect is not included in the linear solution, so there is no response in the x-direction. High damping ratios are included to maintain stability in solving nonlinear equations and for frequency responses about these frequency bands relatively flat.

3.2 Non-linear Solution

The results were obtained by using MATLAB based on ode15s due to its speed in displaying results and good accuracy. Frequency domain and time domain analysis gives valuable information on the evolution of gear conditions and this has been proven in many researches [27][28][29][30]. Figures 7 and Figures 8 illustrate vibration response in y and x direction in the time domain. Figures 9 and Figures 10 illustrate the vibration response in the y and x direction in the frequency domain. Due to the effects of resonance, the third harmonic and the second harmonic have higher amplitudes than other harmonicas.

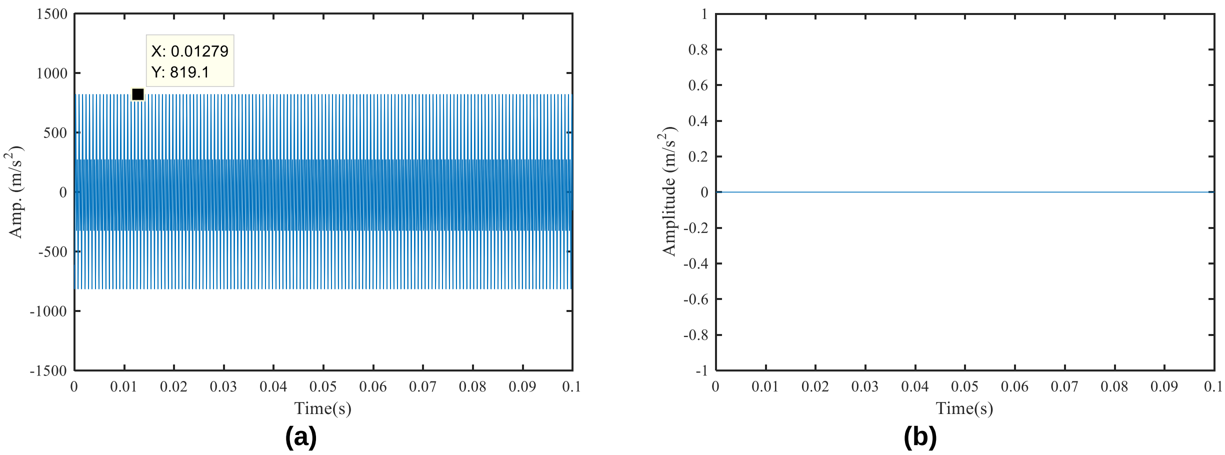
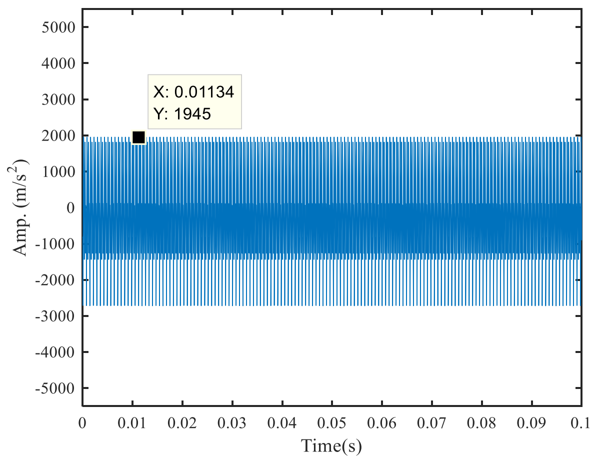
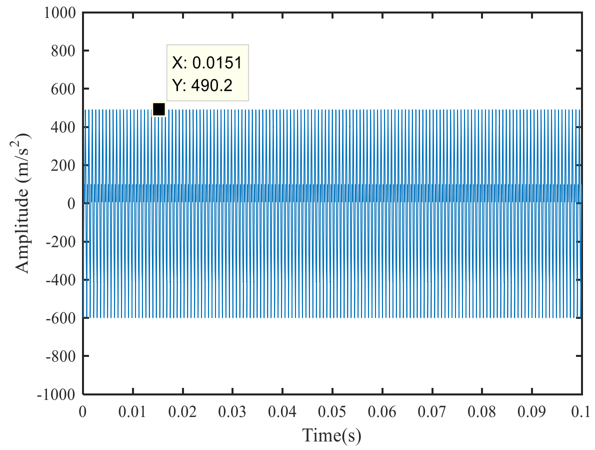


Figure 7. dynamic response in time domain of (without friction) : (a) Y translational of Pinion, (b) X translational of Pinion.



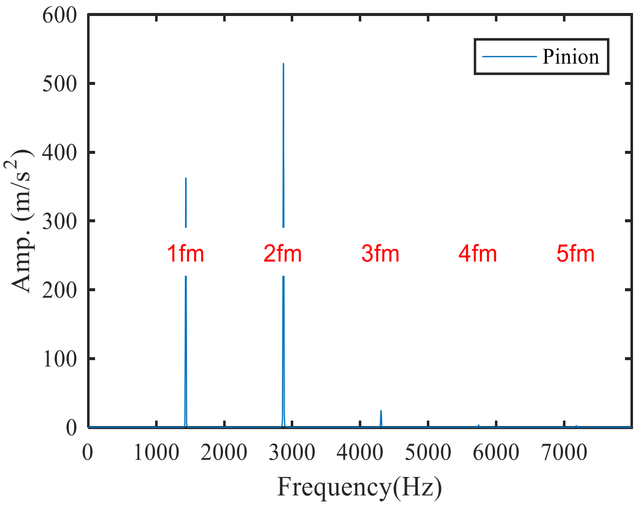
(a)



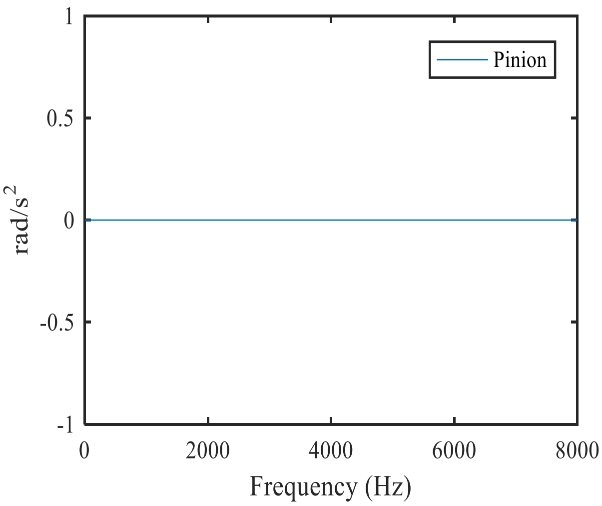
(b)

Figure 8. dynamic response in time domain of (with friction) : (a) Y translational of Pinion, (b) X translational of Pinion.

Figure 7 exposure the vibration signal without adding friction for the mathematical model, as it shows that the vibration amplitude in the Y direction is about 819, while the vibration amplitude in the x direction is equal to zero. Figure 8 exposure the vibration signal with the addition of friction for the mathematical model, as it shows that the vibration amplitude in the Y direction increased to more than two times compared to the vibration amplitude without adding friction. It also exposure that the vibration signal in the X direction increased from zero to 490. The time domain was converted to the frequency domain, depending on the FFT method. Figure 9 exposure the vibration signal without adding friction, as it shows that the GMF amplitude (number of teeth * rotational frequency (Hz)) in Y direction is about 326. Thegear mesh frequency amplitude (GMF) in the X direction is equal to zero.

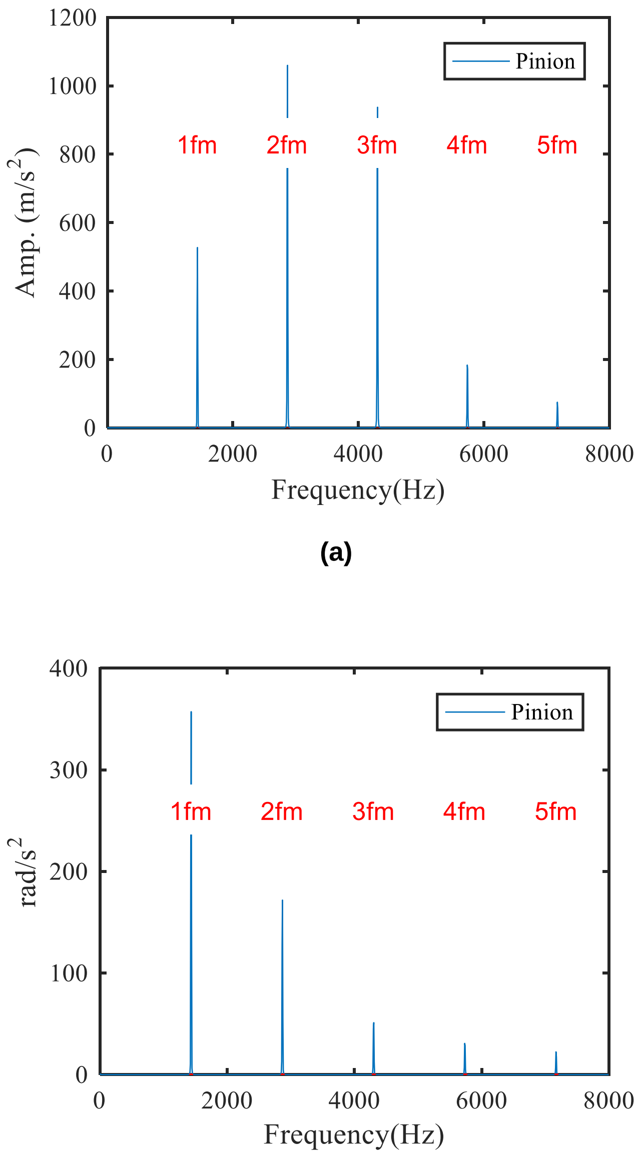


(a)



(b)

Figure 9. dynamic response in frequency domain of (without friction) : (a) Y translational of Pinion, (b) X translational of Pinion.



(a)

(b)

Figure 10. dynamic response in frequency domain of (with friction): (a) Y translational of Pinion, (b) X translational of Pinion.

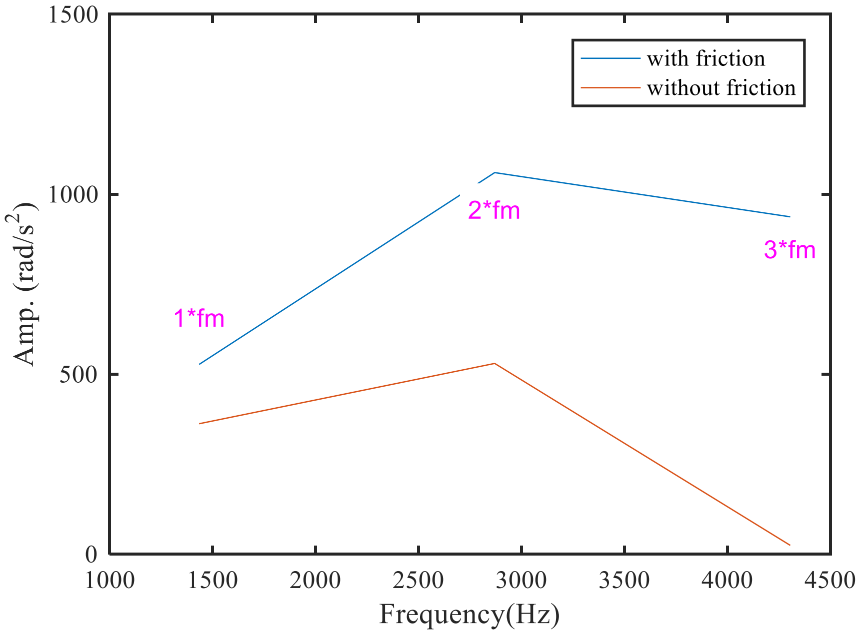


Figure 11. Dynamic response with and without friction for Pinion.

Figure 10 exposure the vibration signal with the addition of friction, as it shows that the GMF amplitude in Y direction increased to 527. It also shows that the gear mesh frequency amplitude in X direction increased from zero to 357.

In addition, the harmonics amplitude of the gear mesh frequency increased significantly and clearly. Figure 11 exposure the gear meshing frequency amplitude and the amplitude of the first and second harmonic without and with the addition of friction. The first harmonic amplitude (2*GMF) in the y-direction increased from 528 to 1060, while its amplitude in the x-direction increased from zero to 171. The second harmonic amplitude (3*GMF) in the y-direction increased from 24 to 937, while its amplitude in the x-direction increased from 0 to 51. From the foregoing, we conclude that friction is a clear and large effect on the vibration response of the gearbox. Therefore, the friction effect must be added to modeling in the best way because that the modeling gives results more close to the real conditions that work in the gearbox.

4. Conclusions

the important sources of vibration is friction between gears, so it must be properly represented in a model. Most researchers neglect the friction effect or consider that the friction effect is constant. This is incorrect because it does not represent the real conditions of the gearbox. The friction effect on the vibration response of gears is demonstrated. Where it was found that friction has a clear effect on the vibration response to gears. Dynamic modeling results indicate that friction increases the vibration amplitude. In addition, it increases the GMF amplitude and increases the amplitude of its harmonics.

Therefore, correctly representing the friction within the modeling improve the representation of the real conditions in which the gearbox are operating.

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