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## ESTIMATING FUZZY HAZARD RATE OF THREE PARAMETERS WEIBULL DISTRIBUTION

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### Abstract

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It is noticed there are many researches and studies in the field of statistics particularly in the field of hazard aiming in order to obtain estimators of very high standard of competency or tainted by instances of uncertainty, which often leads the researcher to estimate the fuzzy hazard functions instead of the usual hazard function, it is the most comprehensive and representative of the data for which the hazard function is to be applied. This paper deals with estimating of fuzzy hazard rate function of three parameters Weibull, methods of Moments Estimators (MOE), maximum likelihood estimators (MLE), and Regression method. The comparison is done by simulation method using different sample size ( $n=40, 60, 80$ ) and initial values of  $(b, c, \delta)$  ( $b$ : scale,  $c$ : shape,  $\delta$ : location parameter). after the parameters are estimated by different three methods (MLE, MOM, PEC(Regression)) ; the values of  $(t_i)$  is generated from C.D.F, using inverse transformation. set of five values of  $(t_i)$  have been taken for application and estimation process. The goal of this process is to get the lowest value of the Mean square error, many fidelity criteria have been computed and the results have been discussed.

**Keywords:** Fuzzy hazard rate, Weibull Distribution, Maximum Likelihood Estimator (MLE), Moments estimator (MOM), Percentiles.

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## Introduction:

Weibull distribution received much interest in reliability theory, and it is used to describe real phenomena and modeling distribution of breaking strength of materials, and also it is used to fit tree diameters data, were this data play an important role in stand model. This distribution was introduced by BAILEY and DELL (1973)(1) as a model for diameter distribution, and have been applied extensively in forestry, since it have ability to describe the wide range of Uniondale distributions. also LITTLE and KILKKI et al (1989)( 8), also Cran in (1988)( 4) work on moment estimators for three parameter weibull distribution, while Tsionas. E. G. in (2000)(13) introduced posterior analysis, prediction and reliability in three parameter weibull distribution. also weibull distribution used to describe real phenomena and modeling distribution of breaking strength of materials as indicated by Johnson et al (1994)( 7),and also murthy et al (2004)( 11). in (2000)( 13) Tsionas, work on estimating Bayesian and also prediction and reliability in three parameters weibull. and Tian. Gives inference about coefficient of variation, in (2015) ( 12)a group of researcher introduced the transmuted exponential weibull distribution with application were this distribution is flexible and useful for analyzing positive data that have bathtub shaped hazard rate function. the obtained new distribution can be used for modeling positive data in various fields like physical and biological sciences, Reliability theory hydrology , medicine and survival analysis and engineering. also in (2014) ( 5) Felix Noyanim and et al introduce comparison between different methods for estimating parameters of weibull, using mean square error as a measure for comparison. in (2013) (10), Mahdi Teimouri and Arjunk. Gupta gives more attention for estimating reliability function for three parameters weibull, and variation and maximum likelihood estimate, and they indicate to Cran (1988) ( 4), when he estimated three parameters by moments of weibull.

## Theoretical Part:

We know that Weibull distribution arises from exponential distribution, when we assume the (pth) power of failure time is random variable have an exponential distribution (i.e) if

$y = T^r$  is r.v ~negative exponential with mean  $\theta$  then

$$g(y, \theta) = \frac{1}{\theta} e^{-\frac{y}{\theta}} \quad y, \theta > 0 \quad \dots\dots(1)$$

Then the failure time ( $T$ ) is r.v. have Weibull distribution (WEb) and its failure rate proportional to some power  $t$ , i.e

$$h(t) = \alpha t^k$$

$$h(t) = c t^k$$

$$\text{Let: } k = p - 1 \quad \text{and } c = \frac{p}{\theta}$$

Then  $T$  is r.v  $\sim$  WEb

$$f(t) = \frac{p}{\theta} t^{p-1} e^{-\frac{t^p}{\theta}} \quad p, \theta > 0 \quad \dots\dots(2)$$

We can prove that

$$\mu'_r = E(t^r) \text{Forf}(t) \text{ in equation (1)}$$

$$\text{Equal } \mu'_r = E(t^r)$$

$$= \theta^{\frac{r}{p}} \Gamma(1 + \frac{r}{p}) \quad \dots\dots(3)$$

$$\text{When } r=1 \rightarrow E(T) = \theta^{\frac{1}{p}} \Gamma(1 + \frac{1}{p})$$

And variance of (T) is

$$\sigma^2_T = E(T^2) + (ET)^2$$

$$\sigma^2_T = \theta^{\frac{2}{p}} \left[ \Gamma\left(1 + \frac{2}{p}\right) - \Gamma^2\left(1 + \frac{1}{p}\right) \right] \quad \dots\dots(4)$$

Here our research deals with three parameters Weibull which was introduced by Petrosion (1994), when the failure happens after certain time (i.e)  $t > \delta$ ,  $\delta < t < \infty$ :

$$f_T(t, b, c, \delta) = \frac{c}{b} \left(\frac{t-\delta}{b}\right)^{c-1} e^{-\left(\frac{t-\delta}{b}\right)^c} \quad \dots\dots(5)$$

Where  $\delta$ : is location parameter.

$b$ : is scale parameter.

$c$  : shape parameter.

Corresponding to equation (5), the cumulative distribution function (CDF) is;

$$F_T(t, b, c, \delta) = 1 - e^{-\left(\frac{t-\delta}{b}\right)^c} \quad t \geq \delta \quad \dots\dots(6)$$

And the reliability function is

$$R_T(t) = \Pr(T > t)$$

$$R_T(t) = e^{-\left(\frac{t-\delta}{b}\right)^c} \quad \dots\dots(7)$$

we also prove that the  $r$  th moments formula of three parameters weibull is;

$$\mu_r^* = \sum_{i=0}^r C_i^r b^r \delta^{r-i} \Gamma\left(\frac{r}{c} + 1\right) \quad \dots\dots(8)$$

We can use equation (8) and Solving equation(9) ;

$$\mu_r^* = \frac{\sum_{i=1}^n t_i^r}{n} \quad \text{for } r = 1, 2, 3 \quad \dots\dots(9)$$

to obtain  $\hat{b}_{mom}$ ,  $\hat{c}_{mom}$ ,  $\hat{\delta}_{mom}$

Maximum Likelihood Method Estimation:

Let  $(t_1, t_2, \dots, t_n)$  be ar.s from p.d.f in equation (5), then;

$$L = \prod_{i=1}^n f(t_i, b, c, \delta)$$

$$L = c^n b^{-n} \prod_{i=1}^n \left( \frac{t_i - \delta}{b} \right)^{c-1} e^{-\sum_{i=1}^n \left( \frac{t_i - \delta}{b} \right)^c}$$

$$L = c^n b^{-n(c-1)} \prod_{i=1}^n (t_i - \delta)^{c-1} e^{-\sum_{i=1}^n \left( \frac{t_i - \delta}{b} \right)^c} \dots \dots (10)$$

Taking logarithmic for equation (10) we obtain ;

$$\log L = n \log c - n(c-1) \log b + (c-1) \sum_{i=1}^n \log(t_i - \delta) - b^{-c} \sum_{i=1}^n (t_i - \delta)^c \dots (11)$$

Then from

$$\frac{\partial \log L}{\partial c} = \frac{n}{c} - n \log b + \sum_{i=1}^n \log(t_i - \delta) - \left( b^{-c} \sum_{i=1}^n (t_i - \delta)^c (1) \log(t_i - \delta) + \sum_{i=1}^n (t_i - \delta)^c (-c) b^{-c-1} \right)$$

$$\text{Put } \frac{\partial \log L}{\partial c} = 0$$

$$\frac{n}{\hat{c}} - n \log b + \sum_{i=1}^n \log(t_i - \delta) - b^{-c} \sum_{i=1}^n (t_i - \delta)^c \log(t_i - \delta) - c b^{-c-1} \sum_{i=1}^n (t_i - \delta)^c = 0 \dots (12)$$

Equation (12) Solved numerically to find  $\hat{c}_{MLE}$

Now;

$$\frac{\partial \log L}{\partial b} = \left[ \frac{-nc}{b} + c b^{-c-1} \sum_{i=1}^n (t_i - \delta)^c \right] * b^{c+1} \div c$$

$$n * b^c = \sum_{i=1}^n (t_i - \delta)^c$$

$$b^c = \frac{\sum_{i=1}^n (t_i - \delta)^c}{n} ; \text{ then}$$

$$\hat{b}_{MLE} = \sqrt[c]{\frac{\sum_{i=1}^n (t_i - \delta)^c}{n}} \dots (13)$$

And ;

$$\begin{aligned} \frac{\partial \log L}{\partial \delta} &= (c-1) \sum_{i=1}^n \frac{-1}{(t_i - \delta)} - b^{-c} \sum_{i=1}^n c (t_i - \delta)^{c-1} (-1) \\ &= (c-1) \sum_{i=1}^n (t_i - \delta)^{-1} + c b^{-c} \sum_{i=1}^n (t_i - \delta)^{c-1} = 0 \end{aligned} \dots \dots (14)$$

Solving equation (14) numerically to find  $(\hat{\delta}_{MLE})$

Third Method Regression Estimation:

$$F(t) = 1 - e^{-(\frac{t-\delta}{b})^c}$$

from

$$e^{-(\frac{t-\delta}{b})^c} = 1 - F(t_i)$$

$$e^{-(\frac{t-\delta}{b})^c} = w_i \quad 0 \leq w_i \leq 1 \quad \dots \dots (15)$$

Taking Logarithm for equation (15); then

$$-\left(\frac{t-\delta}{b}\right)^c = \log w_i$$

we assume  $\log w_i = y_i$ , then

$$(-y_i)^{\frac{1}{c}} = \left(\frac{t_i - \delta}{b}\right)$$

$$\hat{t}_i = \left( b (-y_i)^{\frac{1}{c}} + \delta \right)$$

Then estimation by regression obtained from minimizing total sum of squares between;  $t_i$  and  $\hat{t}_i$ , i.e

$$T = \sum_{i=1}^n \left[ t_{(i)} - \left( b (-y_i)^{\frac{1}{c}} + \delta \right) \right]^2$$

$$T = \sum_{i=1}^n \left[ t_{(i)} - b (-y_i)^{\frac{1}{c}} - \delta \right]^2 \quad \dots \dots (16)$$

From  $\frac{\partial T}{\partial b} = 0$  we find

$$\hat{b}_{(Reg)} = \frac{\left( \sum_{i=1}^n t_{(i)} (-y_i)^{\frac{1}{c}} - \hat{\delta} \sum_{i=1}^n (-y_i)^{\frac{1}{c}} \right)}{\left( \sum_{i=1}^n (-y_i)^{\frac{2}{c}} \right)} \quad \dots \dots (17)$$

Also from  $\frac{\partial T}{\partial c} = 0$  we obtain non linear equation, which is ;

$$\sum_{i=1}^n (-1)^{\frac{1}{c}} t_{(i)} y_i \log\left(\frac{1}{y_i}\right) - b \sum_{i=1}^n (-y_i)^{\frac{2}{c}} \log\left(\frac{1}{y_i}\right) + \hat{\delta} \sum_{i=1}^n (-1)^{\frac{1}{c}} y_i \log(y_i^{-1}) = 0 \quad \dots \dots (18)$$

from equation (18) solved numerically To find  $\hat{c}_{(Reg)}$

$$\frac{\partial T}{\partial \delta} = 2 \sum_{i=1}^n (t_{(i)} - b - (y_i)^{\frac{1}{c}} - \delta) (1) = 0$$

$$= \sum_{i=1}^n t_{(i)} - n\hat{b} - \sum_{i=1}^n (y_i)^{\frac{1}{c}} = n\hat{\delta} \quad \div n$$

$$\hat{\delta}_{MLE} = t - \hat{b} - \frac{\sum_{i=1}^n (y_i)^{\frac{1}{c}}}{n} \quad \dots \dots (19)$$

Simulation Procedure:

To find  $(\hat{b}, \hat{c}, \hat{\delta})$  we apply simulation procedure, taking sample size ( $n = 40, 60, 80$ ), and using inverse transformation of (CDF) as ;

$$U_i = F(t_i, \delta, b, c)$$

$$U_i = 1 - e^{-(\frac{t_i - \delta}{b})^c} \quad t_i \leq \delta$$

$$e^{-(\frac{t_i - \delta}{b})^c} = 1 - U_i$$

$$\log(1 - U_i) = -\left(\frac{t_i - \delta}{b}\right)^c \quad 0 \leq U_i \leq 1$$

$$-\log(1 - U_i) = \left(\frac{t_i - \delta}{b}\right)^c$$

Let  $z_i = -\log(1 - U_i)$  and  $w_i = (z_i)^{\frac{1}{c}}$

$$\text{Then } z_i = \left(\frac{t_i - \delta}{b}\right)^c$$

when  $w_i = (z_i)^{\frac{1}{c}}$

$$\text{Then } (z_i)^{\frac{1}{c}} = \frac{t_i - \delta}{b}$$

$$w_i = \frac{t_i - \delta}{b}$$

$$b w_i = t_i - \delta$$

$$t_i = b w_i + \delta \quad t_i \geq \delta$$

Using generated values ( $t_i$ ) for ( $n = 40, 60, 80$ ) we can find Moment (estimators) from equation (9) and maximum likelihood from equations( 12,13,14), and finally we apply Regression analysis of ( $t_i$ ) ( at  $n= 40, 60, 80$  ) to obtain least square estimator for  $(\hat{b}, \hat{c}, \hat{\delta})$ ; from equation (17,18,19)

and the Results of estimation are compared using statistical measure mean square error (MSE), and we explained in the following table we notice some values of  $\hat{h}(t_i)$ , are greater than (1), this indicates that the value of p.d.f at some set is greater than the value of  $R_T(t)$ , at this set.

Table (1)

contains table (1.1), table (1.2), table(1.3),and table(1.4)

Table (1.1) estimator fuzzy hazard rate when (  $c=3$ ,  $b=2.8$ ,  $\zeta=0.5$ ,  $\check{k}=0.3$  )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.1584	0.0994	0.0151	0.0379	0.0007	0.0041	0.0045	mom
	1.1500	0.2548	0.0999	0.0091	0.0379	0.0048	0.0121	0.0122	mom
	1.4500	0.3513	0.1004	0.0332	0.1137	0.0126	0.0202	0.0215	mom
	1.7500	0.4477	0.1009	0.0574	0.1895	0.0241	0.0305	0.0334	mom
	2.0500	0.5441	0.1014	0.0816	0.2653	0.0392	0.0428	0.0478	mom
60	0.8500	0.1584	0.1019	0.0175	0.0202	0.0006	0.0040	0.0038	mom
	1.1500	0.2548	0.1024	0.0094	0.0202	0.0046	0.0120	0.0110	mom
	1.4500	0.3513	0.1029	0.0363	0.0606	0.0123	0.0198	0.0169	mom
	1.7500	0.4477	0.1034	0.0632	0.1010	0.0237	0.0296	0.0240	mom
	2.0500	0.5441	0.1040	0.0901	0.1415	0.0387	0.0412	0.0324	reg
80	0.8500	0.1584	0.1347	0.0422	0.0079	0.0001	0.0027	0.0029	mom
	1.1500	0.2548	0.1357	0.0001	0.0079	0.0028	0.0130	0.0094	mom
	1.4500	0.3513	0.1366	0.0419	0.0236	0.0092	0.0191	0.0113	mom
	1.7500	0.4477	0.1376	0.0840	0.0393	0.0192	0.0265	0.0133	reg
	2.0500	0.5441	0.1386	0.1260	0.0551	0.0329	0.0350	0.0156	reg

Table (1.2) estimator fuzzy hazard rate when (  $c=3$ ,  $b=2.8$ ,  $\zeta=0.5$ ,  $\check{k}=0.6$  )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.3551	0.0878	0.0005	0.0069	0.0143	0.0251	0.0251	mom
	1.1500	0.5480	0.0886	0.0109	0.1315	0.0422	0.0577	0.0568	mom
	1.4500	0.7408	0.0893	0.0224	0.2562	0.0849	0.1032	0.1012	mom
	1.7500	0.9337	0.0901	0.0338	0.3808	0.1423	0.1619	0.1585	mom

	2.0500	1.1265	0.0909	0.0453	0.5054	0.2145	0.2338	0.2284	mom
60	0.8500	0.3551	0.1209	0.0002	0.0008	0.0110	0.0251	0.0250	mom
	1.1500	0.5480	0.1224	0.0446	0.0151	0.0362	0.0507	0.0545	mom
	1.4500	0.7408	0.1239	0.0894	0.0293	0.0761	0.0849	0.0953	mom
	1.7500	0.9337	0.1254	0.1342	0.0436	0.1307	0.1278	0.1474	Mle
	2.0500	1.1265	0.1269	0.1790	0.0578	0.1998	0.1795	0.2108	Mle
80	0.8500	0.3551	0.1491	0.0018	0.0014	0.0085	0.0250	0.0242	mom
	1.1500	0.5480	0.1514	0.0485	0.0260	0.0314	0.0499	0.0347	mom
	1.4500	0.7408	0.1538	0.0988	0.0506	0.0689	0.0824	0.0470	reg
	1.7500	0.9337	0.1561	0.1491	0.0752	0.1209	0.1231	0.0611	reg
	2.0500	1.1265	0.1585	0.1994	0.0999	0.1874	0.1719	0.0772	reg

Table (1.3) estimator fuzzy hazard rate when (  $c=3$ ,  $b=3.2$ ,  $\zeta=0.5$ ,  $\check{k}=0.3$ )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.0926	0.0892	0.0991	0.0669	0.0002	0.0001	0.0004	mle
	1.1500	0.1770	0.0896	0.0647	0.0423	0.0015	0.0025	0.0036	mom
	1.4500	0.2613	0.0900	0.0303	0.0178	0.0107	0.0059	0.0119	mle
	1.7500	0.3457	0.0904	0.0041	0.0068	0.0230	0.0233	0.0130	reg
	2.0500	0.4301	0.0908	0.0385	0.0314	0.0307	0.0230	0.0318	mle
60	0.8500	0.0926	0.0909	0.0581	0.0945	0.0001	0.0002	0.0003	mom
	1.1500	0.1770	0.0913	0.0383	0.0598	0.0015	0.0023	0.0027	mom
	1.4500	0.2613	0.0918	0.0184	0.0251	0.0102	0.0058	0.0112	mle
	1.7500	0.3457	0.0922	0.0014	0.0096	0.0231	0.0129	0.0226	mle
	2.0500	0.4301	0.0926	0.0213	0.0443	0.0304	0.0228	0.0298	mle
	0.8500	0.0926	0.1092	0.0529	0.0781	0.0001	0.0003	0.0001	mom
	1.1500	0.1770	0.1099	0.0345	0.0494	0.0023	0.0041	0.0009	reg

80	1.4500	0.2613	0.1105	0.0161	0.0207	0.0120	0.0045	0.0107	mle
	1.7500	0.3457	0.1112	0.0023	0.0080	0.0110	0.0236	0.0220	mom
	2.0500	0.4301	0.1118	0.0207	0.0367	0.0287	0.0335	0.0203	reg

Table (1.4) estimator fuzzy hazard rate when (  $c=3$ ,  $b=3.2$ ,  $\bar{z}=0.5$ ,  $\check{k}=0.6$  )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.3316	0.0857	0.0009	0.0023	0.0121	0.0219	0.0219	mom
	1.1500	0.5004	0.0865	0.0615	0.0443	0.0343	0.0385	0.0470	mom
	1.4500	0.6691	0.0872	0.1240	0.0863	0.0677	0.0594	0.0816	mle
	1.7500	0.8379	0.0880	0.1865	0.1284	0.1125	0.0849	0.1257	mle
	2.0500	1.0066	0.0887	0.2489	0.1704	0.1685	0.1793	0.1148	reg
60	0.8500	0.3316	0.1106	0.0018	0.0008	0.0098	0.0218	0.0217	mom
	1.1500	0.5004	0.1119	0.1755	0.0156	0.0302	0.0211	0.0416	mle
	1.4500	0.6691	0.1132	0.3492	0.0303	0.0618	0.0679	0.0205	reg
	1.7500	0.8379	0.1145	0.5228	0.0450	0.1047	0.0199	0.1007	mle
	2.0500	1.0066	0.1157	0.6965	0.0598	0.1587	0.0192	0.1399	mle
80	0.8500	0.3316	0.1266	0.0009	0.0009	0.0084	0.0215	0.0215	mom
	1.1500	0.5004	0.1283	0.0172	0.0177	0.0277	0.0167	0.0410	mle
	1.4500	0.6691	0.1300	0.0353	0.0345	0.0581	0.0543	0.0204	reg
	1.7500	0.8379	0.1317	0.0534	0.0513	0.0997	0.0183	0.1007	mle
	2.0500	1.0066	0.1334	0.0715	0.0681	0.1525	0.0149	0.1262	mle

Table (2)

contains table (2.1), table (2.2), table(2.3),and table(2.4)

Table (2.1) estimator fuzzy hazard rate when (  $c=5$ ,  $b=2.8$ ,  $\zeta=0.5$ ,  $\check{k}=0.3$ )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.1543	0.1414	0.0083	0.0017	0.0043	0.0001	0.0047	mle
	1.1500	0.2949	0.1424	0.0054	0.0011	0.0168	0.0047	0.0173	mle
	1.4500	0.4355	0.1435	0.0025	0.0005	0.0171	0.0375	0.0379	mom
	1.7500	0.5762	0.1445	0.0004	0.0002	0.0664	0.0663	0.0373	reg
	2.0500	0.7168	0.1456	0.0033	0.0008	0.0653	0.1018	0.1027	mom
60	0.8500	0.1543	0.1777	0.0115	0.7475e-3	0.0001	0.0041	0.0047	mom
	1.1500	0.2949	0.1794	0.0075	0.4729e-4	0.0165	0.0027	0.0170	mle
	1.4500	0.4355	0.1812	0.0035	0.1983e-3	0.0373	0.0129	0.0379	mle
	1.7500	0.5762	0.1829	0.0005	0.0763e-3	0.0309	0.0663	0.0660	mom
	2.0500	0.7168	0.1846	0.0045	0.3509e-3	0.1015	0.0566	0.1025	mle
80	0.8500	0.1543	0.2129	0.0093	0.2285e-3	0.0047	0.0040	0.0001	reg
	1.1500	0.2949	0.2155	0.0062	0.1446e-3	0.0013	0.0162	0.0167	mom
	1.4500	0.4355	0.2180	0.0030	0.0606e-3	0.0370	0.0095	0.0379	mle
	1.7500	0.5762	0.2205	0.0001	0.0233e-3	0.0253	0.0662	0.0658	mom
	2.0500	0.7168	0.2230	0.0033	0.1073e-3	0.1012	0.0488	0.1025	mle

Table (2.2) estimator fuzzy hazard rate when (  $c=5$ ,  $b=2.8$ ,  $\zeta=0.5$ ,  $\check{k}=0.6$ )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.5527	0.1383	0.0001	0.0002	0.0344	0.0611	0.0611	Mom
	1.1500	0.8340	0.1402	0.0061	0.0040	0.1371	0.0963	0.1378	Mle
	1.4500	1.1152	0.1422	0.0123	0.0078	0.2433	0.1894	0.2453	mle
	1.7500	1.3965	0.1441	0.0185	0.0117	0.3137	0.3798	0.3835	Mom

	2.0500	1.6777	0.1460	0.0247	0.0155	0.5526	0.5465	0.4692	reg
60	0.8500	0.5527	0.1861	0.0002	0.0100	0.0611	0.0269	0.0611	Mle
	1.1500	0.8340	0.1896	0.0066	0.1895	0.1369	0.0830	0.1362	Mle
	1.4500	1.1152	0.1932	0.0131	0.3690	0.1700	0.2429	0.2444	Mom
	1.7500	1.3965	0.1968	0.0196	0.5485	0.3792	0.2878	0.3833	Mle
	2.0500	1.6777	0.2004	0.0260	0.7280	0.4465	0.5456	0.5525	Mom
80	0.8500	0.5527	0.1823	0.0002	0.0000	0.0610	0.0263	0.0611	Mle
	1.1500	0.8340	0.1858	0.0059	0.0009	0.0822	0.1354	0.1360	Mom
	1.4500	1.1152	0.1893	0.0121	0.0018	0.2431	0.1687	0.2434	Mle
	1.7500	1.3965	0.1928	0.0183	0.0027	0.3832	0.3790	0.2866	reg
	2.0500	1.6777	0.1963	0.0245	0.0035	0.5451	0.4390	0.5506	Mle

 Table (2.3) estimator fuzzy hazard rate when (  $c=5$ ,  $b=3.2$ ,  $\zeta=0.5$ ,  $\bar{k}=0.3$  )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.1543	0.1414	0.0083	0.0017	0.0047	0.0043	0.0001	reg
	1.1500	0.2949	0.1424	0.0054	0.0011	0.0047	0.0168	0.0173	Mom
	1.4500	0.4355	0.1435	0.0025	0.0005	0.0375	0.0171	0.0379	Mle
	1.7500	0.5762	0.1445	0.0004	0.0002	0.0373	0.0663	0.0664	Mom
	2.0500	0.7168	0.1456	0.0033	0.0008	0.1018	0.0653	0.1025	Mle
60	0.8500	0.1543	0.1777	0.0115	0.7475e-3	0.0041	0.0001	0.0045	Mle
	1.1500	0.2949	0.1794	0.0075	0.4729e-3	0.0165	0.0027	0.0170	Mle
	1.4500	0.4355	0.1812	0.0035	0.1983e-3	0.0373	0.0129	0.0375	Mle
	1.7500	0.5762	0.1829	0.0005	0.0763e-3	0.0309	0.0663	0.0664	Mom
	2.0500	0.7168	0.1846	0.0045	0.3509e-3	0.1020	0.1015	0.0566	reg
	0.8500	0.1543	0.1881	0.0074	0.5662e-3	0.0002	0.0038	0.0042	Mom
	1.1500	0.2949	0.1900	0.0049	0.3582e-3	0.0022	0.0162	0.0165	Mom

80	1.4500	0.4355	0.1919	0.0023	0.1502e-3	0.0373	0.0119	0.0371	Mle
	1.7500	0.5762	0.1938	0.0002	0.0578e-3	0.0292	0.0661	0.0664	Mom
	2.0500	0.7168	0.1957	0.0027	0.2658e-3	0.1010	0.0543	0.1018	Mle

Table (2.4) estimator fuzzy hazard rate when ( c=5, b=3.2,  $\zeta=0.5$ ,  $\check{k}=0.6$ )

n	t <sub>i</sub>	h(t)	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.5527	0.1392	0.0001	0.0002	0.0342	0.0611	0.0611	Mom
	1.1500	0.8340	0.1412	0.0085	0.0040	0.0960	0.1363	0.1391	Mom
	1.4500	1.1152	0.1432	0.0172	0.0077	0.2411	0.1890	0.2486	Mle
	1.7500	1.3965	0.1452	0.0259	0.0115	0.3757	0.3132	0.3898	Mle
	2.0500	1.6777	0.1471	0.0346	0.0152	0.5400	0.4686	0.5528	Mle
60	0.8500	0.5527	0.1866	0.0002	0.0076	0.0611	0.0610	0.0268	reg
	1.1500	0.8340	0.1902	0.0071	0.1444	0.0829	0.1361	0.1378	Mom
	1.4500	1.1152	0.1937	0.0139	0.2812	0.2408	0.1698	0.2453	Mle
	1.7500	1.3965	0.1973	0.0207	0.4180	0.3753	0.2876	0.3832	Mle
	2.0500	1.6777	0.2009	0.0275	0.5548	0.4362	0.5322	0.5526	Mom
80	0.8500	0.5527	0.1889	0.0006	0.0000	0.0610	0.0265	0.0610	Mle
	1.1500	0.8340	0.1926	0.0103	0.0003	0.1357	0.0823	0.1365	Mle
	1.4500	1.1152	0.1964	0.0211	0.0006	0.1689	0.2394	0.2451	Mom
	1.7500	1.3965	0.2002	0.0320	0.0009	0.3830	0.3724	0.2862	reg
	2.0500	1.6777	0.2039	0.0428	0.0012	0.4344	0.5312	0.5521	Mom

Table (3)

contains table (3.1), table (3.2), table(3.3),and table(3.4)

Table (3.1) estimator fuzzy hazard rate when (  $c=5$ ,  $b=2.8$ ,  $\zeta=0.8$ ,  $\check{k}=0.3$  )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.1365	0.1611	0.0343	0.0016	0.0020	0.0021	0.0037	Mom
	1.1500	0.2972	0.1625	0.0222	0.0010	0.0176	0.0158	0.0036	reg
	1.4500	0.4579	0.1638	0.0102	0.0004	0.0401	0.0173	0.0419	Mle
	1.7500	0.6186	0.1652	0.0019	0.0002	0.0411	0.0761	0.0768	Mom
	2.0500	0.7793	0.1666	0.0139	0.0007	0.1172	0.0751	0.1212	Mle
60	0.8500	0.1365	0.2023	0.0476	0.7108	0.0025	0.0009	0.0036	Mle
	1.1500	0.2972	0.2046	0.0309	0.4497	0.0017	0.0151	0.0173	Mom
	1.4500	0.4579	0.2068	0.0143	0.1886	0.0419	0.0394	0.0126	reg
	1.7500	0.6186	0.2091	0.0023	0.0725	0.0760	0.0335	0.0765	Mle
	2.0500	0.7793	0.2113	0.0190	0.3336	0.1156	0.0645	0.1210	Mle
80	0.8500	0.1365	0.2423	0.0239	0.2054	0.0001	0.0016	0.0036	Mom
	1.1500	0.2972	0.2455	0.0158	0.1299	0.0005	0.0142	0.0170	Mom
	1.4500	0.4579	0.2488	0.0076	0.0545	0.0388	0.0087	0.0419	Mle
	1.7500	0.6186	0.2521	0.0006	0.0210	0.0763	0.0758	0.0269	reg
	2.0500	0.7793	0.2554	0.0088	0.0964	0.1204	0.1152	0.0549	Mle

Table (3.2) estimator fuzzy hazard rate when (  $c=5$ ,  $b=2.8$ ,  $\zeta=0.8$ ,  $\check{k}=0.6$  )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.4005	0.1539	0.2330	0.0030	0.0122	0.0056	0.0321	Mle
	1.1500	0.7219	0.1564	0.0932	0.0011	0.0640	0.0791	0.1042	Mom
	1.4500	1.0434	0.1589	0.0466	0.0007	0.1565	0.1987	0.2177	Mom
	1.7500	1.3648	0.1614	0.1864	0.0026	0.2896	0.2777	0.3725	Mle

	2.0500	1.6862	0.1640	0.3262	0.0045	0.4635	0.3699	0.5685	Mle
60	0.8500	0.4005	0.2049	0.2755	0.1314	0.0077	0.0031	0.0316	Mle
	1.1500	0.7219	0.2096	0.1064	0.0499	0.0525	0.0758	0.1039	Mom
	1.4500	1.0434	0.2142	0.0628	0.0317	0.1375	0.1923	0.2174	Mom
	1.7500	1.3648	0.2189	0.2319	0.1133	0.2626	0.2567	0.3711	Mle
	2.0500	1.6862	0.2236	0.4011	0.1949	0.4279	0.5656	0.3303	reg
80	0.8500	0.4005	0.2008	0.4318	0.6643	0.0080	0.0002	0.0314	Mle
	1.1500	0.7219	0.2053	0.1770	0.2520	0.1035	0.0594	0.0534	reg
	1.4500	1.0434	0.2099	0.0778	0.1603	0.1389	0.1865	0.2172	Mom
	1.7500	1.3648	0.2145	0.3326	0.5727	0.2647	0.2131	0.3708	Mle
	2.0500	1.6862	0.2190	0.5874	0.9850	0.4305	0.2415	0.5648	Mle

Table (3.3) estimator fuzzy hazard rate when (  $c=5$ ,  $b=3.2$ ,  $\zeta=0.8$ ,  $\check{k}=0.3$  )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.0078	0.1272	0.0406	0.0053	0.0068	0.0004	0.0001	Reg
	1.1500	0.1484	0.1281	0.0341	0.0044	0.0026	0.0004	0.0044	Mle
	1.4500	0.2891	0.1290	0.0275	0.0036	0.0137	0.0051	0.0167	Mle
	1.7500	0.4297	0.1299	0.0209	0.0027	0.0180	0.0334	0.0369	Mom
	2.0500	0.5703	0.1308	0.0143	0.0018	0.0386	0.0618	0.0650	Mom
60	0.8500	0.0078	0.1773	0.0542	0.3271	0.0057	0.0002	0.0001	Reg
	1.1500	0.1484	0.1791	0.0455	0.2731	0.0002	0.0021	0.0041	Mom
	1.4500	0.2891	0.1810	0.0367	0.2191	0.0127	0.0023	0.0163	Mle
	1.7500	0.4297	0.1828	0.0280	0.1651	0.0122	0.0323	0.0365	Mom
	2.0500	0.5703	0.1846	0.0192	0.1110	0.0607	0.0297	0.0649	Mle
	0.8500	0.0078	0.1928	0.0697	0.7447	0.0023	0.0002	0.0001	Reg
	1.1500	0.1484	0.1951	0.0584	0.6217	0.0001	0.0016	0.0040	Mom

80	1.4500	0.2891	0.1974	0.0471	0.4987	0.0117	0.0017	0.0160	Mle
	1.7500	0.4297	0.1997	0.0358	0.3758	0.0106	0.0310	0.0362	Mom
	2.0500	0.5703	0.2020	0.0245	0.2528	0.0596	0.0271	0.0645	Mle

 Table (3.4) estimator fuzzy hazard rate when ( c=5, b=3.2,  $\zeta=0.8$ ,  $\check{k}=0.6$ )

n	t <sub>i</sub>	h(t)	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.4063	0.1302	0.0109	0.0058	0.0313	0.0152	0.0391	Mle
	1.1500	0.6875	0.1319	0.0043	0.0022	0.0617	0.0934	0.0945	Mom
	1.4500	0.9688	0.1337	0.0024	0.0014	0.1877	0.1868	0.1395	reg
	1.7500	1.2500	0.1355	0.0090	0.0050	0.3100	0.3080	0.2484	reg
	2.0500	1.5312	0.1373	0.0156	0.0086	0.3886	0.4594	0.4688	Mom
60	0.8500	0.4063	0.1625	0.0546	0.1817	0.0119	0.0247	0.0386	Mom
	1.1500	0.6875	0.1654	0.0220	0.0689	0.0886	0.0545	0.0939	Mle
	1.4500	0.9688	0.1683	0.0105	0.0439	0.1836	0.1281	0.1872	Mle
	1.7500	1.2500	0.1712	0.0431	0.1566	0.2328	0.2913	0.3097	Mom
	2.0500	1.5312	0.1741	0.0756	0.2694	0.4238	0.3684	0.4637	Mle
80	0.8500	0.4063	0.1952	0.1952	0.0249	0.0110	0.0089	0.0295	Mle
	1.1500	0.6875	0.1995	0.1995	0.0106	0.0538	0.0476	0.0917	Mle
	1.4500	0.9688	0.2038	0.2038	0.0038	0.0442	0.1170	0.1862	Mom
	1.7500	1.2500	0.2082	0.2082	0.0182	0.3035	0.2171	0.1579	reg
	2.0500	1.5312	0.2125	0.2125	0.0326	0.2716	0.3478	0.4292	Mom

Table (4)

contains table (4.1), table (4.2), table(4.3),and table(4.4)

Table (4.1) estimator fuzzy hazard rate when (  $c=3$ ,  $b=2.8$ ,  $\zeta=0.8$ ,  $\check{k}=0.3$ )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.0329	0.1002	0.4426	0.1682	0.0055	0.0945	0.0081	Mom
	1.1500	0.0635	0.1007	0.3708	0.1404	0.0010	0.0589	0.0012	Mom
	1.4500	0.1599	0.1013	0.2989	0.1127	0.0007	0.0139	0.0004	Reg
	1.7500	0.2564	0.1018	0.2271	0.0849	0.0048	0.0020	0.0059	Mom
	2.0500	0.3528	0.1023	0.1553	0.0571	0.0125	0.0078	0.0275	Mle
60	0.8500	0.0329	0.1134	0.6543	0.0919	0.0043	0.0423	0.0031	Reg
	1.1500	0.0635	0.1142	0.5501	0.0768	0.0005	0.0474	0.0001	Reg
	1.4500	0.1599	0.1149	0.4459	0.0616	0.0004	0.0133	0.0019	Mom
	1.7500	0.2564	0.1156	0.3416	0.0464	0.0040	0.0015	0.0045	Mle
	2.0500	0.3528	0.1163	0.2374	0.0312	0.0112	0.0027	0.0227	Mle
80	0.8500	0.0329	0.1337	0.4942	0.0487	0.0035	0.0356	0.0013	Reg
	1.1500	0.0635	0.1347	0.4157	0.0406	0.0003	0.0248	0.0001	Mom
	1.4500	0.1599	0.1358	0.3373	0.0326	0.0001	0.0123	0.0002	Mom
	1.7500	0.2564	0.1368	0.2588	0.0246	0.0029	0.0010	0.0032	Mle
	2.0500	0.3528	0.1379	0.1804	0.0165	0.0092	0.0017	0.0206	Mle

Table (4.2) estimator fuzzy hazard rate when (  $c=3$ ,  $b=2.8$ ,  $\zeta=0.8$ ,  $\check{k}=0.6$ )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.2403	0.1454	0.1952	0.0296	0.0033	0.0026	0.0089	Mle
	1.1500	0.4332	0.1478	0.0891	0.0112	0.0205	0.0237	0.0356	Mom
	1.4500	0.6260	0.1502	0.0169	0.0072	0.0553	0.0742	0.0766	Mom
	1.7500	0.8189	0.1526	0.1230	0.0255	0.0988	0.0969	0.1259	Mle

	2.0500	1.0117	0.1550	0.2290	0.0439	0.1468	0.1225	0.1873	Mle
60	0.8500	0.2403	0.1115	0.3508	0.0302	0.0024	0.0022	0.0088	Mle
	1.1500	0.4332	0.1128	0.1417	0.0114	0.0163	0.0170	0.0356	Mom
	1.4500	0.6260	0.1141	0.0675	0.0073	0.0524	0.0634	0.0761	Mom
	1.7500	0.8189	0.1155	0.2767	0.0260	0.0981	0.0588	0.1257	Mle
	2.0500	1.0117	0.1168	0.4858	0.0447	0.1402	0.0553	0.1870	Mle
80	0.8500	0.2403	0.1300	0.1263	0.0214	0.0018	0.0004	0.0078	Mle
	1.1500	0.4332	0.1319	0.0521	0.0081	0.0157	0.0167	0.0351	Mom
	1.4500	0.6260	0.1338	0.0222	0.0052	0.0485	0.0629	0.0756	Mom
	1.7500	0.8189	0.1356	0.0965	0.0185	0.0934	0.0444	0.1243	Mle
	2.0500	1.0117	0.1375	0.1708	0.0318	0.1399	0.0414	0.1796	Mle

 Table (4.3) estimator fuzzy hazard rate when (  $c=3$ ,  $b=3.2$ ,  $\zeta=0.8$ ,  $\bar{k}=0.3$  )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.0047	0.0812	0.4008	0.1008	0.0025	0.0614	0.0018	Reg
	1.1500	0.0891	0.0816	0.3364	0.0842	0.0001	0.0164	0.0010	Mom
	1.4500	0.1734	0.0819	0.2720	0.0675	0.0017	0.0077	0.0054	Mom
	1.7500	0.2578	0.0823	0.2076	0.0509	0.0062	0.0005	0.0086	Mle
	2.0500	0.3422	0.0826	0.1432	0.0342	0.0135	0.0079	0.0190	Mle
60	0.8500	0.0047	0.1038	0.5352	0.0244	0.0020	0.0563	0.0008	Reg
	1.1500	0.0891	0.1044	0.4491	0.0204	0.0001	0.0159	0.0009	Mom
	1.4500	0.1734	0.1050	0.3631	0.0164	0.0009	0.0072	0.0049	Mom
	1.7500	0.2578	0.1056	0.2770	0.0123	0.0046	0.0003	0.0021	Mle
	2.0500	0.3422	0.1062	0.1909	0.0083	0.0111	0.0046	0.0123	Mle
	0.8500	0.0047	0.1088	0.0928	0.0581	0.0022	0.0016	0.0006	Reg
	1.1500	0.0891	0.1094	0.0783	0.0485	0.0001	0.0022	0.0003	Mom

80	1.4500	0.1734	0.1101	0.0639	0.0389	0.0008	0.0024	0.0036	Mom
	1.7500	0.2578	0.1108	0.0494	0.0293	0.0043	0.0001	0.0004	Mle
	2.0500	0.3422	0.1115	0.0349	0.0197	0.0106	0.0035	0.0108	Mle

Table (4.4) estimator fuzzy hazard rate when ( $c=3$ ,  $b=3.2$ ,  $\delta=0.8$ ,  $\bar{k}=0.6$ )

n	$t_i$	$h(t)$	$\hat{h}_{mom}$	$\hat{h}_{mle}$	$\hat{h}_{reg}$	Mse-mom	Mse-mle	mse-reg	Best
40	0.8500	0.2438	0.0836	0.1230	0.0589	0.0051	0.0029	0.0068	Mle
	1.1500	0.4125	0.0843	0.0517	0.0223	0.0215	0.0260	0.0304	Mom
	1.4500	0.5813	0.0851	0.0197	0.0142	0.0492	0.0631	0.0643	Mom
	1.7500	0.7500	0.0858	0.0911	0.0507	0.0882	0.0887	0.0978	Mom
	2.0500	0.9187	0.0865	0.1624	0.0873	0.1385	0.1144	0.1383	Mle
60	0.8500	0.2438	0.0964	0.1236	0.0735	0.0043	0.0029	0.0058	Mle
	1.1500	0.4125	0.0974	0.0528	0.0279	0.0199	0.0259	0.0296	Mom
	1.4500	0.5813	0.0984	0.0180	0.0177	0.0630	0.0466	0.0635	Mle
	1.7500	0.7500	0.0994	0.0889	0.0634	0.0847	0.0874	0.0943	Mom
	2.0500	0.9187	0.1004	0.1597	0.1090	0.1339	0.1132	0.1311	Mle
80	0.8500	0.2438	0.1117	0.1243	0.0545	0.0035	0.0029	0.0046	Mle
	1.1500	0.4125	0.1131	0.0614	0.0207	0.0179	0.0247	0.0286	Mom
	1.4500	0.5813	0.1144	0.0014	0.0132	0.0436	0.0626	0.0631	Mom
	1.7500	0.7500	0.1158	0.0643	0.0470	0.0804	0.0868	0.0934	Mom
	2.0500	0.9187	0.1172	0.1271	0.0808	0.1285	0.1123	0.1304	Mle

## Results and Discussion:

In table(1) include four sub tables due to different sets of initial values of parameters ( $b$ ,  $c$ ,  $\delta$ ,  $\bar{k}$ ), and all the results of estimation are compared used Mean square error (MSE), so we find that Moments estimator(MOM), is best for all inputs table (1), with percentage (51%), while (MLE) estimators of fuzzy hazard rate function is preferred with (28%), while the third fuzzy hazard estimator is obtained from regression estimation with percentage (21%).

For main table (2) and according to different results for estimating  $\hat{h}(t_i)$ , we conclude  $\hat{h}(t_i)_{mom}$  is best with percentage (38%), and the  $\hat{h}(t_i)_{mle}$  is best with (48%), and the third one is  $\hat{h}(t_i)_{reg}$  is best with (14%).

Finally for third Main table (3) and according to different results for estimating fuzzy hazard rate function  $\hat{h}(t_i)$ , by three different methods, we conclude the  $\hat{h}(t_i)_{mom}$  is best with (36%), while  $\hat{h}(t_i)_{mle}$  is best (45%), and  $\hat{h}(t_i)_{reg}$  is best with (19%).

And finally for table (4), the results indicates that  $\hat{h}(t_i)_{mom}$  is best with (43%) and  $\hat{h}(t_i)_{mle}$  is best with (43%), finally  $\hat{h}(t_i)_{reg}$  is best with (14%).

Finally the best estimators of  $\hat{h}(t_i)$  is  $\hat{h}(t_i)_{mle}$ , between all alternative, due to its excellent properties, that is satisfied by (MLE) estimators.

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