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# CALCULATIONS OF RADII FOR STRONTIUM ISOTOPES( 78-100SR) USING DEFORMATION COEFFICIENTS

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## Abstract

In this study, to calculate the isotope radii of strontium ( 78-100Sr), the deformation coefficients which depend on (b2,d) and the root mean square radius ( $\langle r^2 \rangle^{1/2}$ )are calculated. The main and secondary elliptical parts( a, b) , with the difference between them (ΔR)are taken. These parameters are calculated for the even-paired of78-100Sr isotopes (Z =38).The low transition probability B(E2) and the deformation parameters δ which in turn depend on the electric quadrupole moments $(Q_0)$ arecalculated using the equations of the deformed coat model. The variety of shapes of the nuclei for the selected isotopes is observed by drawing three-dimensional (axially symmetric) shapes and two-dimensional shapes of strontium isotopes to distinguish between them using quasi-large (a) and quasi-small (b) Axes.

**Keywords**: Deformation, Strontium, Nuclear Radius, Quadrupole Moment.

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## 1. Introduction

The independent-particle (or shell) model is built on the idea of a Fermi gas: nucleons are assumed to be point particles that are free to orbit within the nucleus due to the net attractive force of a potential-well.
The model assumes that the nuclear force acting between nucleons produces a net potential-well that pulls all nucleons toward the center of the nucleus, and not directly toward other individual nucleons. Nucleons under the influence of that potential-well can exist in distinct quantal energy states, the magnitude and occupancy of which determines the detailed nucleon build-up procedure [1].
An analogous phenomenon occurs in nuclear physics. There exist many experimental indications showing that atomic nuclei possess a shell-structure and that they can be constructed, like atoms, by filling successive shells of an effective potential well [2].
Although the liquid drop model of the nucleus has proved to be quite successful for predicting subtle variations in the mass of nuclides with slightly different mass and atomic numbers, it avoids any mention of the internal arrangement of the nucleons in the nucleus. Yet, there are hints of such an underlying structure. It is observed that there is an abnormally high number of stable nuclides whose proton and/or neutron numbers equal the magic numbers [3] .
For stabilized nuclei, the nuclear shape is basically spherical. This is an effort to minimize the surface energy, in a similar to a drop of fluid. However, small leaving from spheres are observed, for example, in the region $150 < A < 190$. One way to quantify these "deformations" is to use the ratio[4]:

$$\delta = \frac{\Delta R}{R} \qquad (1)$$

Where $\Delta R$: is the difference between semi-major and semi-minor axes. For a sphere $\Delta R$ = 0.
$R$: is the average nuclear radius given below.

$$R = r_0 \times A^{1/3} \qquad (2)$$

Where $r_o$ = 1.2 fm, and $A$: is the mass number.

## 2.Nuclear Surface Deformations

The collective motion can be explained as nuclear surface vibrations and rotations in the geometrical collective model that was firstly suggested by Bohr and Mottelson [5], where a nucleus modeled like a charged liquid drop and the moving nuclear surface may be expressed quite generally by an extension in spherical consistent with time-dependent shape parameters that are considered as coefficients [6]:

$$R(\theta, \emptyset, t) = R_{av}[1 + \sum_{\lambda=0}^{\infty} \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}(t) Y(\theta, \emptyset)] \qquad (3)$$

Where:
$R(\theta,\ \phi,\ t)$: Indicates the nuclear radius in the direction $(\theta,\ \emptyset)$ at time t as shown in figure (1), $Rav$: The average nucleus radius. $\alpha$: Are the deformation variables. $\lambda$: determines the multipole or mode of nuclear motion. $\mu$: is the projection of $\lambda$on the z-axis.$Y(\theta,\ \emptyset)$: is the spherical harmonic ..

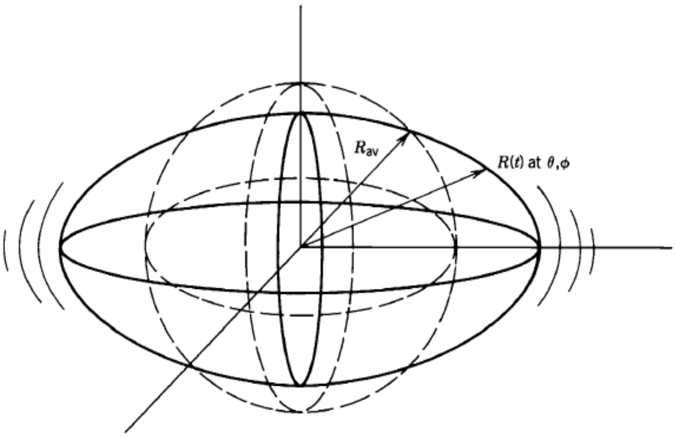


**Figure (1): A vibrating nucleus with a spherical equilibrium shape. The time-dependent coordinate R(t) locates a point on the surface in the direction $(\theta, \emptyset)$[6].**

The quadrupole deformation parameter $\beta_2$, is related to the spheroid [7]:

$$\beta_2 = \frac{4}{3}\sqrt{\frac{\pi}{5}}\frac{\Delta R}{R_{av}} = 1.06\frac{\Delta R}{R_{av}} \qquad (4)$$

Where: The average radius $\mathrm{R_{av}} = \mathrm{R_0 A^{1/3}}$

ΔR: The difference between both of the semi-major and minor axes (omit, repeated). As long as the value of $\beta_2$ is larger, the nucleus becomes more disfigured.The root mean square (rms) radius $\langle r^2 \rangle^{1/2}$,is deduced directly from the distribution of scattered electrons; for a uniformly charged sphere, the squared charge distribution radius $\langle r^2 \rangle$ [8]:

<r2>1/2 =(0.6*(1.2*A(1/3))2*(1+(10/3)*(π*ao/R)2))/(1+(π*ao/R)2) (5)

**3. Electric Quadruple Moment**

The charge allocation in a nucleus can be described in terms of electric multipole moments and pursued from the classical electrostatics thoughts [9]. Several nuclei have constant quadruple moments which may experimentally be measured. These nuclei are expected to have oval shape with a symmetrical axis. This proposition, has classically led to define the intrinsic quadrupole moment as given in the following equation [10]:

$$Q_0 = \int d^3\rho(r)(3z^2 - r^2) \qquad (6)$$

Where

$\rho(r)$: Radial charge density of the proton.

If $Q_0$ is consider to be calculated for a homogeneously charged ellipsoid with charge Ze and semi-axes (a) and (b). With (b) pointing along the z axis, $Q_0$ will be[11]:

$$Q_0 = \frac{2}{5}Z(a^2 - b^2) \qquad (7)$$

If the deviation from sphericity is not very large, the average radius:

$R = 1/2\,(a+b)$ and $\Delta R = (b-a)$, the quadrupole moment is[11]:

$$Q_0 = \frac{4}{5} Z R^2 \delta \qquad (8)$$

The nucleus quadruple distortion parameter values δ calculated from the equation[12]:

$$\delta = 0.75\ Q_0/(Z\langle r^2\rangle) \qquad (9)$$

The semi-axes $(a)$ and $(b)$ are gained from the two following equations [13].

$$a = \sqrt{\langle r^2\rangle(1.66 - \frac{2\delta}{0.9})} \qquad (10)$$

$$b = \sqrt{5\langle r^2\rangle - 2a^2} \qquad (11)$$

**4. Result and Discussion :**

Many parameters are computed which required for the present study, and can be listed as:

- The deformation parameters of the even-even nuclei with mass numbers less than 100 for strontium element .
- The deformation parameters $(\beta_2)$ based on the electric reduced transition probability $B(E2)\uparrow$ values and comparing these values with the deformation Parameters which also calculated from the predicted values of $B(E2)\uparrow$ for (EWSR ) form Raman[13].
- The deformation parameters $(\delta)$ by calculating the intrinsic quadruple moment $(Q_0)$.

1. The Root Mean Square Charge Radius $\langle r^2\rangle^{1/2}$.
2. Semi-major $(a)$ and semi minor $(b)$ axis and the difference between them $(\Delta R)$.
3. All these Values $B(E2)\uparrow,\ Q_0,\ \beta_2, \delta,\ \langle r^2\rangle^{\frac{1}{2}},\ a,\ b, and\ \ \Delta R)$ are represented in tables (1) and (2).
4. The elliptical shapes of Sr isotopes are drawn using semi major and semi minor axes and the difference between them to determine the prolate shapes as shown in fig.(1) .

From table (1) , we observe that the minimum value of deformation $(\beta_2$= 0.1244) corresponds to the value of the first excited energy $2_1^+$ ($E_\gamma = 1836.087\ KeV)$ for strontium-88 and the maximum value of deformation is $(\beta_2 = 0.4119\ KeV)$ corresponds to the value of the first excited energy $2_1^+(E_\gamma = 129.7)$ forstrontium-100 . The other distortion values of the other isotopes vary between these values. This is due to the fact that the lower of first excited State energy $2_1^+$, the greater the reduced transition probability $B(E2)$. Thus, the value of deformation $\beta_2$ increases.As well as the values of the deformation Parameters $\delta$ of the same isotopes, table (1), since the increase in the number of nucleons outside the closed shell leads to increased polarization effect and thus increase the electric quadrupole moments $(Q_0)$ and then increase the deformation ratio of the nucleus .The Mean Square Charge Radius $\langle r^2\rangle$ which is obtained from equation (5) for A < 100 .Intrinsic Quadrupole Moments $(Q_0)$ of nuclei were calculated from the equation (8). These values previewed in tables from ( 2) and compared with the other values of $Q_0$ that were obtained from the EWSR (P.W.).Semi-Major and Semi-Minor axis (a, b) and the deference between them $\Delta R$:And these values previewed in the tables (2) .

**5. Conclusion:**

Determination the value of electric quadrupole moment (Qo) (charge distribution) of the nucleus for determination the shape of the nucleus and the amount of deformation happening in it. The value of deformation parameter (β2) (shape of nuclide) depending generally on the electric quadrupole moment (Qo) of the nucleus. A nucleus with magic number of neutrons (fill shell) leads to a spherical equilibrium of its charge division and spherical profile of nuclei. This proves that less deformation occurs in the shape of nuclei when its neutrons number approach to the magic neutrons number and it increases when step up away. It turns out that different isotopes of strontium, which have a magic number of neutrons, have very small deformation coefficients, so the shape of the nuclei is close to spherical and more stable.

Table (1): Isotopes Mass Number $(A)$, Neutron Number $(N)$, Gamma Energy of the First Excited State $2_1^+(E_\gamma)$, Nuclear Average Radius ($R_0^2$), Reduced Electric Transition Probability $(B(E2)\uparrow$ in unit of $e^2b^2$, Quadrupole Moment $(Q_o)$ in unit of barn, and Deformation Parameters $(\beta_2, \delta)$ for strontium.

| Z | A | N | $E_\gamma(KeV)$ | Theoretical Value | | Present Work | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | $B(E2)\uparrow$ $(e^2b^2)$ for $(EWSR)$ | $\beta_2$ for $(EWSR)$ $(P.w.)$ | $R_0^2$ | $B(E2)\uparrow$ $(e^2b^2)$ | $Q_o$ (b) | $\beta_2$ | $\delta$ |
| 38 | 78 | 40 | 278.5 | 1.08 | 0.4358 | 5.1272 | 0.7384 | 2.7246 | 0.3603 | 0.264 |
| | 80 | 42 | 385.86 | 0.959 | 0.4038 | 0.2674 | 0.5241 | 2.2953 | 0.2935 | 0.2194 |
| | 82 | 44 | 573.54 | 0.513 | 0.2905 | 0.2718 | 0.3468 | 1.8672 | 0.2388 | 0.1761 |
| | 84 | 46 | 793.3 | 1.289 | 0.4531 | 0.2762 | 0.2467 | 1.5750 | 0.1983 | 0.1466 |
| | 86 | 48 | 1076.68 | 0.128 | 0.1406 | 0.2806 | 0.2806 | 1.3414 | 0.1662 | 0.1233 |
| | 88 | 50 | 1836.087 | 0.092 | 0.1174 | 0.2849 | 0.1034 | 1.0193 | 0.1244 | 0.0925 |
| | 90 | 52 | 831.680 | 0.113 | 0.1281 | 0.2892 | 0.2248 | 1.5032 | 0.1807 | 0.1348 |
| | 92 | 54 | 814.98 | 0.114 | 0.1268 | 0.2935 | 0.2260 | 1.5075 | 0.1786 | 0.1336 |
| | 94 | 56 | 836.91 | 0.118 | 0.1272 | 0.2977 | 0.2170 | 1.4770 | 0.1725 | 0.1294 |
| | 96 | 58 | 814.93 | 0.240 | 0.1789 | 0.3019 | 0.2197 | 1.4863 | 0.1712 | 0.1287 |
| | 98 | 60 | 114.225 | 1.282 | 0.4078 | 0.3061 | 1.2247 | 3.5088 | 0.3985 | 0.3005 |
| | 100 | 62 | 129.7 | 1.42 | 0.4234 | 0.3102 | 1.3436 | 3.6752 | 0.4119 | 0.3113 |

Table (2): The number mass $(A)$, The number neutron $(N)$, Root Mean Square Radii $<r^2>^{1/2}$, Minor and Major axes(b,a) and the difference between them (Δ R) by two method for strontium Isotopes.

| Z | A | N | Present Work | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | $\langle r^2\rangle^{1/2}$ fm | a (fm) | b (fm) | $\Delta R_1$ | $\Delta R_2$ | $\Delta R_3$ |
| 38 | 78 | 40 | 4.5136 | 2.2080 | 3.5802 | 1.2067 | 1.3722 | 1.7481 |
| | 80 | 42 | 4.5443 | 2.3149 | 3.4648 | 1.0114 | 1.499 | 1.4602 |
| | 82 | 44 | 4.5746 | 2.4154 | 3.3474 | 0.8187 | 0.9320 | 1.1782 |
| | 84 | 46 | 4.6043 | 2.4847 | 3.2672 | 0.6871 | 0.7825 | 0.9858 |
| | 86 | 48 | 4.6336 | 2.5403 | 3.2034 | 0.5824 | 0.6632 | 0.8330 |
| | 88 | 50 | 4.6625 | 2.6100 | 3.1126 | 0.4405 | 0.5027 | 0.6282 |
| | 90 | 52 | 4.6909 | 2.5323 | 3.2602 | 0.6465 | 0.7279 | 0.9195 |
| | 92 | 54 | 4.7190 | 2.5424 | 3.2661 | 0.6454 | 0.7237 | 0.9154 |
| | 94 | 56 | 4.7466 | 2.5586 | 3.2620 | 0.6295 | 0.7034 | 0.8904 |
| | 96 | 58 | 4.7739 | 2.5673 | 3.2692 | 0.6306 | 0.7019 | 0.8898 |
| | 98 | 60 | 4.8008 | 2.1899 | 3.7964 | 1.4823 | 1.6065 | 2.0862 |
| | 100 | 62 | 4.8274 | 2.1695 | 3.8372 | 1.5459 | 1.6677 | 2.1705 |

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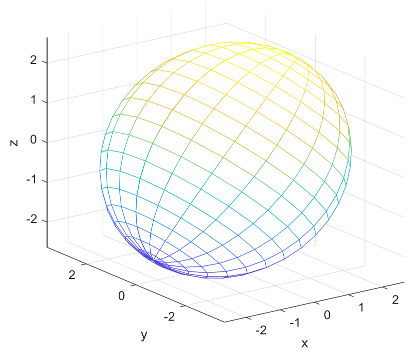
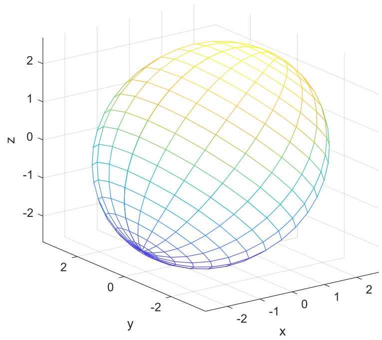
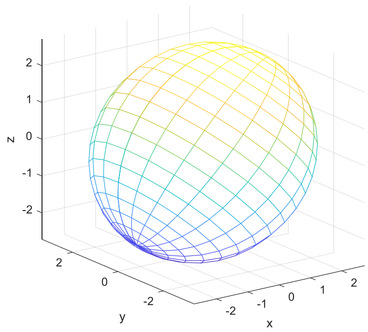


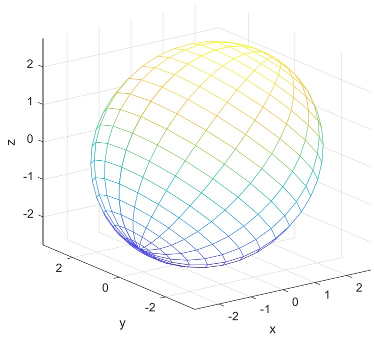
78Sr



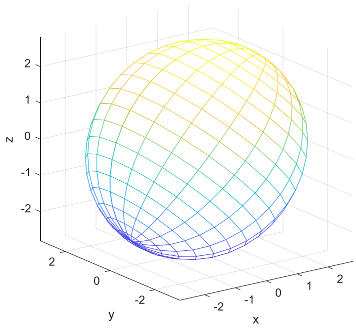
80Sr



82Sr



84 Sr

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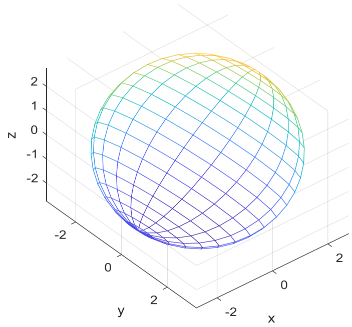
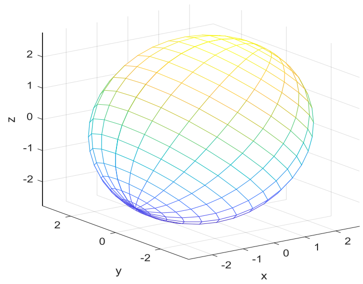
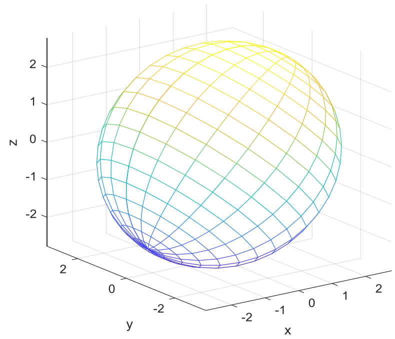


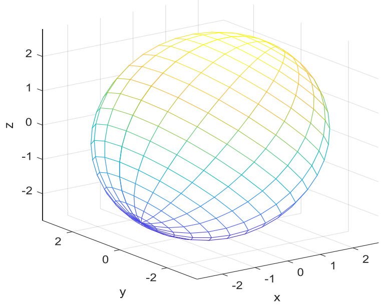

88 Sr



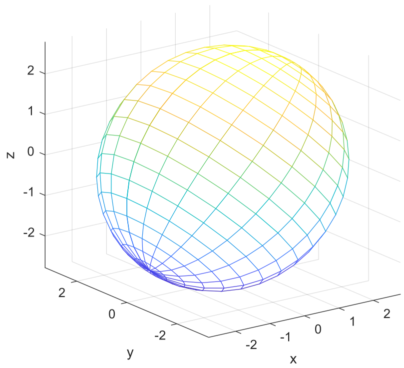

90Sr




92 Sr




94 Sr

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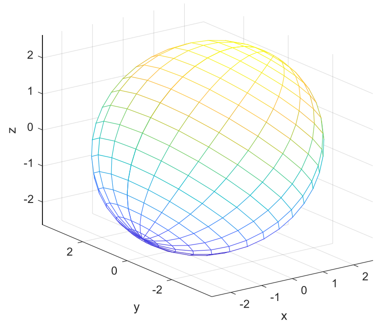
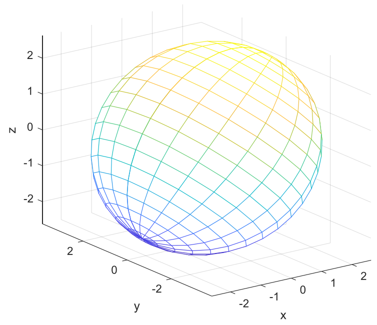



98 Sr




100Sr

Figure (1): 3-D Shapes of axially symmetric quadrupole deformation for strontium Isotopes from major (a) and minor (b) axes.