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ANALYSIS AND EMPLOYING OF HUNGARIAN ALGORITHM USING KRUSKAL'S METHOD FOR ASSIGNMENT PROBLEM

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Abstract

The assignment problem is important matter in the field of information technology, because It treats with assigning one of methods of managing a set of works estimated as (n) that are Symbolized (W1, W2, W3,…..Wn) by using one of available machines that are Symbolized (M1, M2, M3,…..Mn), with minimum cost which is counted by (∑Cij). There are several methods handling this case such as Hungarian's method. In this paper, will been use graphs by converting an assignment into a complete bipartite graph using Kruskal's Algorithm in order to find the minimum spanning tree, and obtaining the most optimal assignment. The results of various cases were compared, and the results were satisfying in both methods, whereas the method Kruscal's algorithm was easiest to have optimal solution.

Keywords: Kruskal's Method, Hungarian Algorithm, Assignment.

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1. Introduction

The problem of assignment is important in the practical field, especially in establishments. Its importance lies in the existence of several works estimated at (n) which is possible to implement each one by using any available facilities such as machines or workers estimated as (n), that differ from each other in terms of cost, time or the proficiency of accomplishing each work, then the required is choosing one of available and appropriate facilities to implement each work by lowest possible cost and time, with a high proficiency in achievement. [1, 2, 3, 4].

It consists of set of management's machines (n), symbolized as follows: (M1, M2, M3, …..Mn). Moreover, there are a set of works (n) symbolized as (W1, W2, W3, …..Wn), that required to be assigned to these machines [5]. The cost of achieving any work via one of machines, will be symbolized as Cij (If completing a particular work via a machine was possible Cij will have high value.) [6]. So any machine can perform only one work, therefore the problem is assigning works to the machines, so that the total cost of performing a work would be at a minimum [7]. To phrase this matter mathematically, until the following is defined:

$$X_{ij} = \begin{cases} 1 & \text{If the job j to the machine i} \\ 0 & \text{In the remained cases} \end{cases}$$

Since any machine fits only to one work:

$$\sum_{j=1}^{n} X_{ij} = 1;$$

i= 1,2,3,………..,n

Every work is doled out to special case machine:

$$\sum_{i=1}^{n} X_{ij} = 1;$$

j=1,2,3,……….,n

Thus, the objective function takes the following pattern:

$$\min Z = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij}\, X_{ij}$$

1. Literature Survey

Herein it will be try to explain the historical background of assignment problem, and the scientists who devised or developed a method to solve this problem, during consecutive periods, starting from oldest to the latest: First it should be mentioned that the scientist (Carl Gustav Jacobi) had solved the assignment problem since 19th century, and published it in 1890 in Latin. This was discovered just in 2006. After that, in 1931 the German Scientist of mathematics (Konig), has developed a method of solving assignment problem, this method was known as (Hungarian), and this is a skillful method to find an optimal solution [8]. In this developed method the scopes of costs offer the results that related with assigning a resource to an action as objecting to the value of the biggest or smallest amount of cost's assignment [9]. in addition, if minify the matrix of cost to be containing at least one zero in each row and column, in this case it will be able to make an optimal assignment when the scope of cost is equal to zero [10]. The Hungarian method is a combinatorial algorithm that can solve the assignment problem in multinomial time and can expect duple essential methods. Also in (1955), the scientist (Kuhn) composed an assignment issue which became known later as "Hungarian technique". It was used mostly by the two Hungarian mathematicians: (Denes Kınig) and (Jenı Egervary) to solve urgent issues. This algorithm was reviewed by the scientist (James Munkres) in (1957), he noted that it is very multinomial. Since that, the algorithm became known as (Kuhn Munkres) algorithm. Then the scientists (Ford and Fulkerson) used this algorithm to solve the general transportation's problems. Further, in 1981 A new method of assignment problems was put by the mathematician

(Thompson), this method is using methodical way to deal with assignment problem with considering each row at a time and finally moving forward until determining optimal solution in any row. In (1990) (Nagaraj Balakrishnan) has suggested a new method to obtain an optimal solution. In addition to the mathematician (Ahmed, afaq Ahmed) who has developed an algorithm to find the optimal solution to the assignment problem. In 1995 the mathematicians (Li) and (Smith) proceeded their study on traffic flow system using randomly overcrowding, finally they put an algorithm for quadratic Assignment Problems, which is used to solve complicated problems in this issue [11]. Moreover, in 1997 the alternative of Hungarian Method was concluded by mathematician (Ji), this method is based on 2n × 2n matrix levels wherein operation is performed till reaching an optimal solution. In the period between the years 2004 until 2007, the scientists (Charalampos Papamanthou), (Anshuman Sahu), and (Rudrajit Tapadar), have presented further studies in this field. In the following will be mention some studies of distinguished mathematicians:1. Assignment in terms of one's – by Shweta Singh,G.C.Dubey, Rajesh Shrivastava (2012). 2. Profit association with optimal workers to job assignment pattern – by Elsiddigidriss Mohamed Idriss (2013). 3. For the unbalanced problems, space required for Hungarian to solve is n2 – by Jameer, G. Kotwal, Tanuja S.Dhope (2015) [12]. In this paper, will been use graphs by converting an assignment into a complete bipartite graph using Kruskal's Algorithm in order to find the minimum spanning tree, and obtaining the most optimal assignment.

2. Hungarian Method

The Hungarian method is one of the best methods and that used to solve the problem of assignment in polynomial time [13]. This method has been developed to solve the problem of assignment sufficiently by depending on the mathematical property discovered by the Hungarian scientist, Konig (Who the method was named by his name). Through this method, assuming that the values of costs (Cij) are not negative [14]. The fundamental principle of this method stipulates that the subtraction or addition of a constant in any row or column in the matrix, do not effect on the optimal assignment. If the cost of any work achieved via the machine 1, for instance, if has been contracted on (k) dollar, the objective function of assignment will be as follows [15, 16]. The following objective function, required to be at a minimum:

$$\min Z = \sum_{j=1}^{n}(C_{1j} - K)\, X i j + \sum_{i=2}^{n}\sum_{j=1}^{n} C_{ij} X_{ij} \dots\dots\dots (1)$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij} X_{ij} - K\sum_{j=1}^{n} X\, ij$$

However, $\sum_{j=1}^{n} X_{1j} = 1$ because only one work is assigned to the machine 1, so that the function Z would be:
Z = (the original value) – K
The solution is to subtract the largest cost from various rows and columns, so that ideal assignment will be obtained through inspection. The algorithm is performed by examining each row and column in the cost's matrix, in order to identify the minimum cost. Then, the obtained quantity should be subtracted from all rows or columns, so that is getting a cost's matrix comprising at least one non-existent element from each row (or column). After that, try carrying out the assignment by using no-cost cells, and if it is achieved, optimal assignment can be obtained [17, 18]. Such an assignment has non-negative costs (Cij), so the minimum value of the objective function (iΣj Cij Xij) cannot be less than zero. Thus, the assignment is achieved depending on low costs.

3. Proposed Algorithm

Before explaining the algorithm, must be identify some concepts related to graph:
The first concept: Graph (G) is defined as a set of vertices (V) called nodes or points, which are connected by edges (E) with unordered pairs of vertices, the elements of (E) and (V) are called "a graph".
The second concept: Complete multiple edge graph: a graph is called as a complete multiple edges, if the graphs are divided into two partial groups (V1), (V2), that is, each node in V1 is close to node in (V2), each node in (V2) is next to the node in (V1), and there are no two close nodes in same partial graph (V1) or (V2) [19].
The third concept: Tree: A tree is known as both connected and acyclic graph.

3.1. Issues of Assignment and Data

It can be said that the data theory is wonderful fundamental themes in modern mathematics, as this theory is employed in most fields of knowledge. Also it is considered to be a simple mathematical sample for each system, including assignment issues [20], for instance, with a note that assignment meets the issue of data, in other words, any assignment's issue can be represented in a graph, and the result is a complete multiple edge graph (see Fig.1).

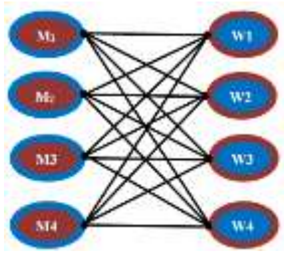

Fig. 1 (Complete Bipartite Graph)

Hence, according that, any assignment issue can be represented graphically by following this method.

3.2. Minimum Spanning Trees

The total weight of a tree T is meant the sum of edge-weighted graphs T. If symbolized them as μ (T), it would be:

$$\mu\,(T) = \sum_{e_i \in E(T)} \mu(e_i) \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

As E (T) is the sum of edges T, and μ (ei) is the weight of the edge (ei).

3.3. Kruscal's for Minimum Spanning Tree

In 1956, Kruscal developed a brief method to find the minimum spanning tree. It is summarized as follows [22,23]:

Start with an edge from the minimum-weighted graph (G); such as edge e1, then take the other edges successively (e2 , e3 ,., en-1) provided that you choose in each stage an edge that is not chosen before, and with the most minimum weight in comparison with the remained edges in G, with considering it should not constitute a circuit with some chosen edges. Thus, the partial graph T consisting of the chosen edges by this method e1, e2, …, en-1 would be a tree generating to the most minimum weight G.

3.4. Employing Kruscal's Method for Assignment

In this section, will be deal with the matter of assignment by employing a new method which is similar to Kruscal's method but providing additional conditions for this method. So, will be rephrase the method as follows:

- First step: Representing data graphically, the graph will be (G).
- Second step: The edge of the graph G which has the most minimum weight such as the edge (e1) will be located in the point Mi, for instance.

Third step: Edges will be chosen from the point Mi only, and the conditions of choosing are:

a) An edge that is not chosen before,
b) Its size is smaller than or equal to the size of the remained edges in G.
c) It does not form a circuit with some chosen edges.

Fourth step: If there is no edge (or completing the test of all edges) in the vertex Mi regarding the aforementioned conditions, an edge will be choose from another vertex with regarding same conditions, and same steps will be repeated with all remained edges in G until the partial graph T comprising edges (e1, e2 ,…, en-1) chosen by this method become a generated tree of G, which is the most total minimum.

Fifth step: Can now achieve an effective assignment, to do so, firstly begin to assign the first-class vertices, then will be assign the highest vertices, considering that each Machine can be assigned to one work only. By doing that, the optimal assignment will be achieved.

In first example, need to find the optimal assignment for four machines, and the cost of assignment is illustrated in Table 1:

Table 1 The Cost of the Assignment for four Machines

	W_1	W_2	W_3	W_4
M_1	10	9	8	7
M_2	3	4	5	6
M_3	2	1	1	2
M_4	4	3	5	6

The solution (Hungarian method). Since the number of works is equal to the number of machines, therefore it is the standard assignment. Firstly, will be begin to examine the rows, then will be conclude that a matrix of reduced cost

can be obtained by doing the followings:

1. Subtract the value 7 from the first row.
2. Subtract the value 3 from the second row.
3. Subtract the value 1 from the third row.

4- Subtract the value 3 from the fourth row.

Thus, will be get the matrix of cost as shown in Table 2:

Table 2 Matrix of low cost

	W_1	W_2	W_3	W_4
M_1	3	2	1	10
M_2	0	1	2	3
M_3	1	0	0	1
M_4	1	0	2	3

From the previous table, an effective assignment was reached by using cells which have no cost only, and it is equivalent to M4 -W2, M3 -W3, M2 -W1, M1 -W4. Hence, this assignment becomes optimal, and the total cost is 7+3+1+3=14.

It is not always possible to find an effective assignment by using cells with no cost in general. To clarify that, will be take the following assignment as the second example to find the perfect solution for the question of assignment by the matrix of the cost as shown in Table 3:

Table 3 Assignment problem of cost

	W_1	W_2	W_3	W_4
M_1	10	9	7	8
M_2	5	8	7	7
M_3	5	4	6	5
M_4	2	3	4	5

The solution (Hungarian method). Firstly, the smallest element is subtracted in each row from the remained elements of that row. This leads to the matrix of low cost as shown in Table 4:

Table 4 Matrix of low cost

	W_1	W_2	W_3	W_4
M_1	3	2	0	1
M_2	0	3	2	2
M_3	1	0	2	1
M_4	0	1	2	3

As the two machines M1, M2 has no cost for the work W1 only; it is not possible to consider that the assignment which depends on no-cost cells only, is effective. To get more zeros, the lowest element in the fourth column should be subtracted from all elements of that column as shown in Table 5:

Table 5 After lowest element in the fourth column is subtracted from all elements of that column in table 4.

	W_1	W_2	W_3	W_4
M_1	3	2	0	0
M_2	0	3	2	1
M_3	1	0	2	0
M_4	0	1	2	2

By using no-cost cells, it is possible to assign only three works. So, an effective assignment cannot be achieved. In such cases, will be represent a minimum number by drawing a line passing through some rows and columns (vertically or horizontally) in order to cover the whole no-cost cells. The number of low numbers required for that, is the total number of works which can be assigned by using those cells.
In the following example, it is achieved by drawing three lines as shown in Table 6:

Table 6 After lowest element in the fourth column is subtracted from all elements of that column in table 4.

	W_1	W_2	W_3	W_4
M_1	3	2	0	0
M_2	0	3	2	1
M_3	1	0	2	0
M_4	0	1	2	2

Now, will be choose the minimum value from uncovered elements. In above example, will be choose the element 1, then subtract this value from all uncovered elements, also this value will be added to whole covered elements located in the intersection of any two lines. Thus, will be obtain a matrix with the following reduced cost as shown in Table 7:

Table 7 This is obtained after the selection of minimum values from uncovered elements,

	W_1	W_2	W_3	W_4
M_1	4	2	0	0
M_2	0	2	1	0
M_3	2	0	2	0
M_4	0	0	1	1

The latter procedure is equivalent to subtracting the element 1 from the two lines, second and fourth, and adding it to the first column in the cost matrix. Therefore, once again, the optimal assignment does not change.
Now, it is probable to perform an effective assignment, and that assignment will be optimal if accomplishing the followings:

$M_4 \rightarrow W_2$,

$M_3 \rightarrow W_4$,

the total cost is 7+5+5+3=20. If an effective assignment is not achieved in this stage, lines should be drawn again to cover zeros, and this method should be repeated until achieving an effective assignment.

Now will be repeat practicing the two mentioned examples by using the proposed algorithm. According to the first example as shown in Table 8:

Table 8 Applying the algorithm to the first example.

	W_1	W_2	W_3	W_4
M_1	10	9	8	7
M_2	3	4	5	6
M_3	2	1	1	2
M_4	4	3	5	6

By applying the algorithm steps to the example (see Fig.2), it would be:

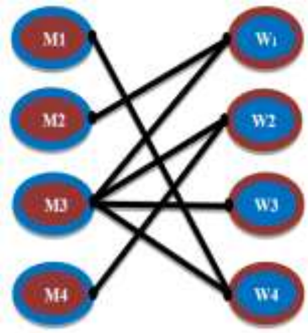

Fig. 2 Applying the proposed algorithm

With respect to weights, it is fixed in the matrix above. The second step: will be start with the edge M3 W2 because its size is smaller than others (it is also possible to start with the edge M3 W3), then choose M3 W1, M3 W4, M3 W3. Now have two choices, choosing either M2 W1, or M4 W2.
Suppose that it is M2 W1, and then choose M4 W2. And continue applying the algorithm steps, the result will be (see Fig.3):

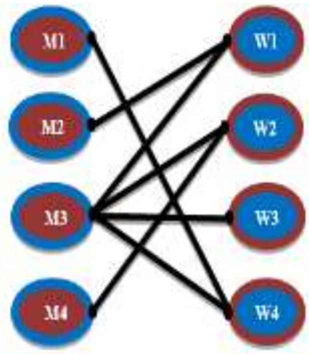

Fig. 3 Applying the algorithm to second step.

Now, it is possible to achieve an effective assignment, and will been notice that the degree of the vertex M3 is 4 According to other vertices, there is only one edge that connecting those vertices with the group W. Thus; the assignment is performed by vertices M1, M2, M4 firstly, then by the vertex M3 as it is illustrated in the following form:

$M_3 \rightarrow W_3$,

$M_4 \rightarrow W_2$,

This is the most optimal assignment which will be also obtain when using the previous method. The total cost is 1+3+3+7=14.
According to the second example:

Table 9. Applying the algorithm to second example

	W_1	W_2	W_3	W_4
M_1	10	9	7	8
M_2	5	8	7	7
M_3	5	4	6	5
M_4	2	3	4	5

When applying the algorithm, the starting point is in the edge of M4 W1. Moreover, the graph resulted in the algorithm will be (see Fig.4):

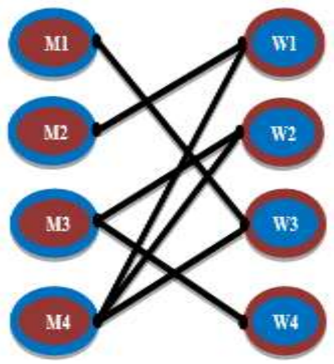

Fig. 4 After Applying the proposed algorithm

The assignment would be firstly

$M_1 \rightarrow W_3 \qquad M_2 \rightarrow W_1$

Because the degree of both vertices is M1, M2. According to vertex M4, it will note that there are three joint works. Since each work is assigned to only one machine, and W3, W1 are assigned. Thus the work W2 is assigned to the machine M4. Finally, have only one choice in terms of the work W4, which is assigned to the machine M3. So, the assignment will be as follows:

$M_4 \rightarrow W_2$,

$M_2 \rightarrow W_1$,

This is the most optimal assignment as it is obtained from the previous method. The total cost is 5+3+5+7=20.

4. Conclusions

According to what is mentioned, conclude the following:
Accessing the most optimal assignment by employing those two methods is easy.
Graphs were used employed to handle the issue of assignment by proposing a new appropriate algorithm. Thus the most optimal assignment is found efficiently.

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